JOURNAL OF THE AMERICAN

ROCKET

SOCIETY

A journal devoted to rocket technology and the jet propulsion sciences

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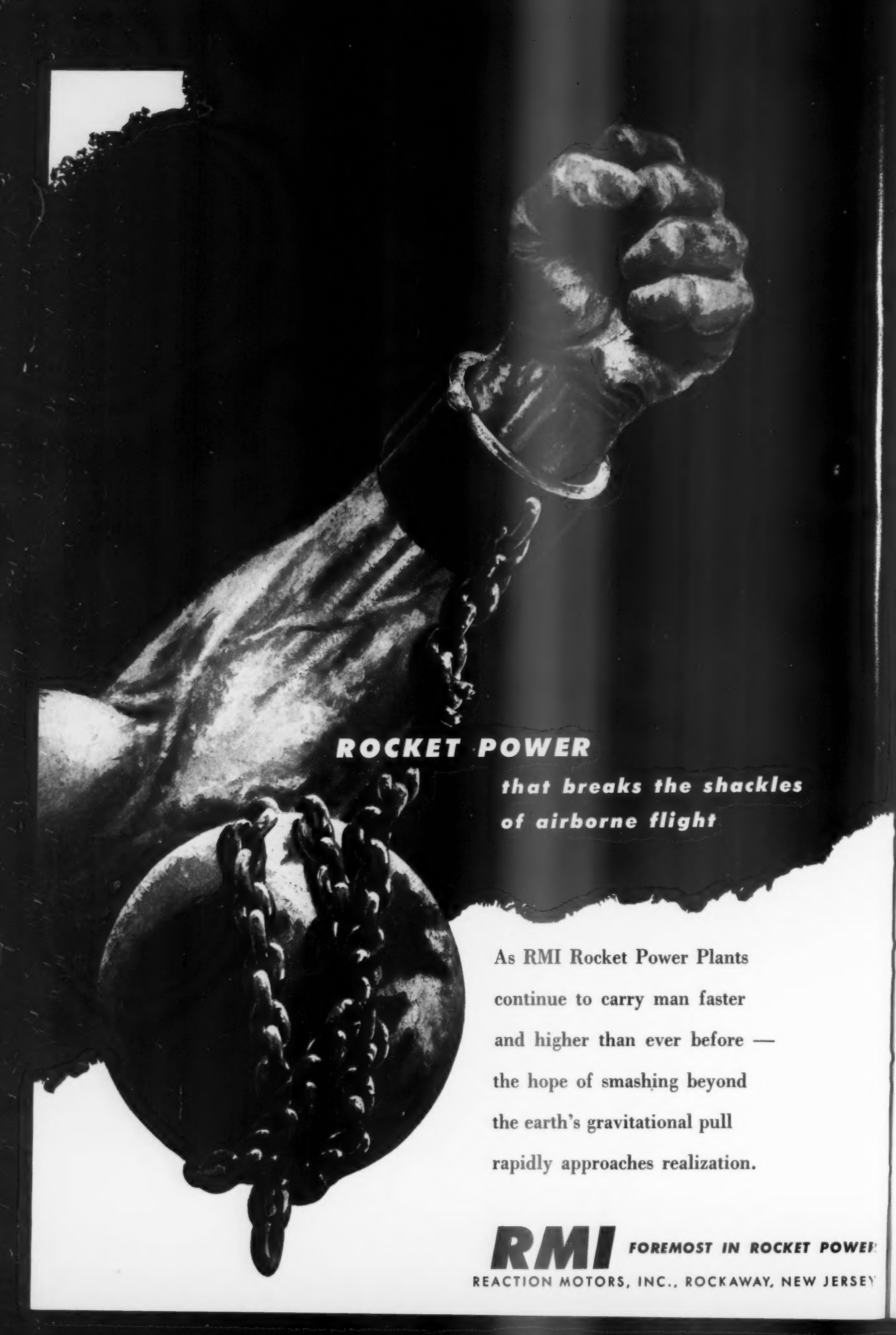

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The Journal of the American Rocket Society is devoted to the advancement of the field of jet propulsion through the publication of original papers disclosing new knowledge and new developments. The term "jet propulsion" as used herein is understood to embrace all engines that develop thrust by rearward discharge of a jet through a nozzle or duct, and thus includes systems utilizing atmospheric air and underwater systems, as well as rocket engines. The Journal is open to contributions, either fundamental or applied, dealing with specialized aspects of jet and rocket propulsion, such as fuels and propellants, combustion, heat transfer, high temperature materials, mechanical design analyses, flight mechanics of jet-propelled vehicles, astronautics, and so forth. The Journal endeavors, also, to keep its subscribers informed of the affairs of the Society and of outstanding events in the rocket and jet propulsion field.

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Manuscripts should be submitted in duplicate to the Editor-in-Chief, Martin Summerfield, Department of Aeronautical Engineering, Princeton University, Princeton, N. J. See instructions on the inside back cover.

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The September 1951 issue was the first of a new series of expanded scope and contents. This series is numbered on an annual volume basis, beginning each year with the January issue as Number 1. Correspondingly, the September issue was Number 5, and since 1951 was the 21st year of publication the new series started with Volume 21 Number 5. This replaced Number 86 of the previous series.

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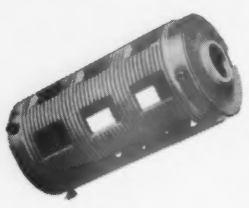

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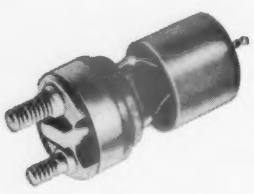

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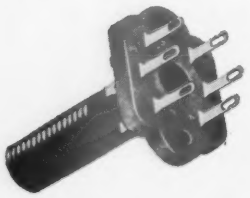

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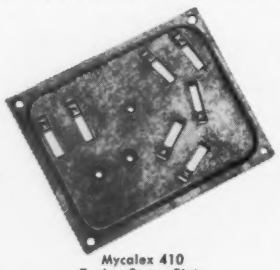

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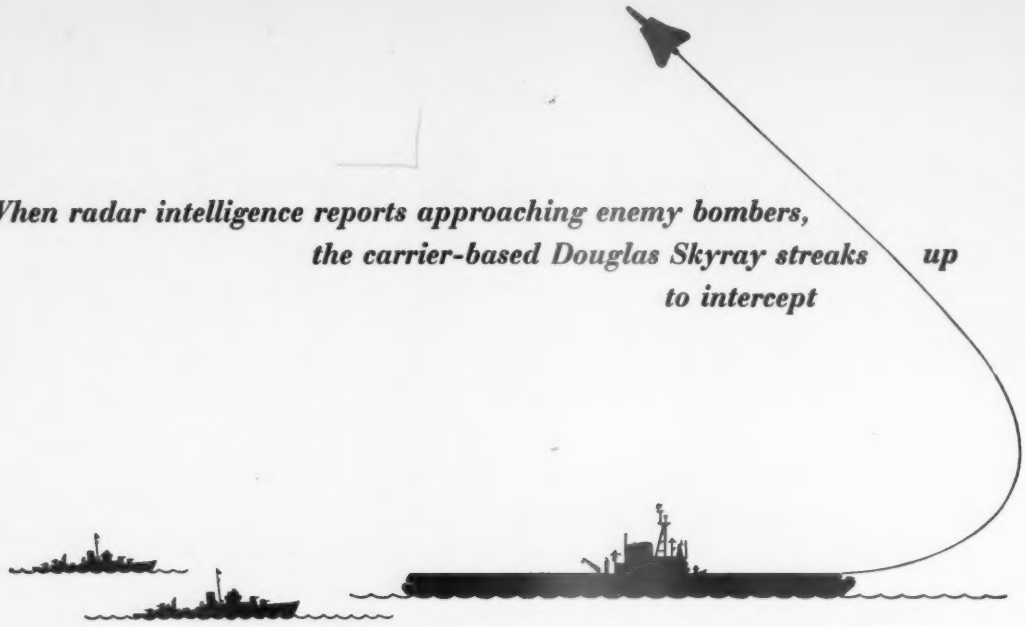

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JOURNAL OF THE AMERICAN ROCKET SOCIETY

Editor-in-Chief
MARTIN SUMMERFIELD

An Editorial

This issue is the sixth of the new series of the JOURNAL OF THE AMERICAN ROCKET SOCIETY that began with the September-October issue of 1951. It marks the end of one year of development of the new JOURNAL. It constitutes an appropriate occasion, therefore, for the Editor-in-Chief, who was assigned the responsibility for organizing the new series, to discuss with its readers the present position and future prospects of the JOURNAL.

Our achievements can be summarized in a few brief statements. In the past year, by virtue of the quality of its contents, the JOURNAL undoubtedly has earned a more advanced position among the scientific periodicals of the world. A high technical level in keeping with the concepts of the jet propulsion profession has been established through careful adherence to rigorous yet open-minded standards of manuscript acceptance. A steadily increasing number of excellent technical articles are being offered for publication, and a comfortable but not excessive backlog of accepted papers now exists. Two new sections, Jet Propulsion News and Technical Literature Digest, have been introduced which have proved of strong interest to many readers.

The growth in content and the elevation of standards have been accompanied by marked advances on other sides. The circulation of the JOURNAL has climbed steeply in the past year. This issue is being mailed to fifty per cent more names than the corresponding issue a year ago. Perhaps more significant is the increase in the number of institutional libraries that have purchased subscriptions. An author seeking to place his paper before the largest possible group of readers competent in jet propulsion will find it now to his advantage to offer his manuscript to the J.A.R.S.

Financially, the JOURNAL has made striking progress in the past twelve months. Advertising income has increased eightfold on an annual basis. The advertisers include many of the most distinguished business organizations and institutions in the United States. Their participation is not only the result of solicitation by the Society but reflects as well their own recognition of the importance of the new JOURNAL.

All of this growth has been accomplished not without bringing to the fore numerous problems, some minor and some major. Two of the most important deserve mention here, since they will affect the future character of the JOURNAL.

One of these stems from the existence of military security restrictions throughout many of the areas of jet propulsion work. In the present unfortunate state of world tension the editors feel it their duty to maintain proper secrecy, and accordingly it is our practice to require that a manuscript be properly cleared if the subject matter warrants it. Uncertainty has arisen in some quarters as to the proper function of an open scientific journal that finds itself limited thus by security restrictions. Is it supposed to content itself with semitechnical articles that give information of only general character, or is there a more serious function it can perform? We are convinced that there is.

Our technical strength as a nation depends not only on current work on better weapons carried on in secret, but even more in the long range on our capacity for new developments. This capacity is stimulated largely by open exchange of basic ideas among fertile minds in diverse fields, and by the transmission of such ideas to students in colleges and graduate schools. This free exchange is possible only through open publication. The editors hope that potential contributors will offer to this JOURNAL their scientific advances whenever this can be done without jeopardizing military security.

The second problem concerns the scope of subject matter of the JOURNAL. Historically the Society and its JOURNAL were founded to promote interest in extra-terrestrial flight, and from the beginning it was natural for most of the early contributors to focus their attention on rocket power plant development. In recent years, however, the membership of the Society has broadened in character and now includes many professional engineers interested in terrestrial application of rocket propulsion, in the nonpropulsive aspects of rocket flight, and more significantly, in other forms of jet propulsion. It has become clear that the rocket motor is not greatly different from the ramjet, for example, and that the fundamental processes in each (combustion, gas dynamics, etc.) are very much related. Also, the power plant is no longer a more important factor in high-speed vehicles than, for example, the aerodynamic configuration or the navigational system. Therefore, it seemed timely last year to take steps to correct this early impression of the JOURNAL's scope, without departing from our vital interest in space flight.

A review of the past six issues will reveal that a broader policy was established not only in the acceptance of contributed articles but also in the contents of the Jet Propulsion News and Technical Literature Digest sections. This policy will be continued. It is our hope that it will meet with approval among potential contributors who may have felt formerly that the JOURNAL was concerned narrowly with rocket motors.

Martin Summerfield
Professor of Jet Propulsion
Princeton University

Automatic Navigation of a Long Range Rocket Vehicle

H. S. TSIEN,[1] T. C. ADAMSON,[2] and E. L. KNUTH[2]

Daniel and Florence Guggenheim Jet Propulsion Center, California Institute of Technology, Pasadena, Calif.

The flight of a rocket vehicle in the equatorial plane of a rotating earth is considered with possible disturbances in the atmosphere due to changes in density, in temperature, and in wind speed. These atmospheric disturbances together with possible deviations in weight and in moment of inertia of the vehicle tend to change the flight path away from the normal flight path. The paper gives the condition for the proper cut-off time for the rocket power, and the proper corrections in the elevator angle so that the vehicle will land at the chosen destination in spite of such disturbances. A scheme of tracking and automatic navigation involving a high-speed computer and elevator servo is suggested for this purpose.

THE behavior of a vehicle flying through air is closely dependent upon the aerodynamic forces acting upon the vehicle. If, during one period of oscillation of the vehicle, there is appreciable variation of the response of aerodynamic forces to the attitude of the vehicle through variations in speed, in aerodynamic coefficients, in air density, etc., then the behavior of the disturbed flight path cannot be described by a linear differential equation of constant coefficients. In fact, the basic differential equation actually has coefficients that are specified functions of time. A very simple example of such motion is that of an artillery rocket during burning of the propellant grain. As shown by J. B. Rosser, R. R. Newton, and G. L. Gross (1),[3] the basic differential equation for this particular case can be written as Bessel's differential equation for the order 1/2. The general character of the solutions of such differential equations is quite different from the character of solutions of differential equations with constant coefficients. For instance, while for equations with constant coefficients the stability of solutions for the homogeneous equation is generally sufficient to insure the stability of solutions with reasonable forcing functions, this simple state of affairs no longer prevails for equations with variable coefficients. The present theory of control and stability is built almost exclusively upon the theory of differential equations with constant coefficients. Therefore to study the disturbed motion of rockets, new methods have to be used.

R. Drenick in a recent paper (2) demonstrated the usefulness of ballistic disturbance theory in solving the control and guidance problem of ballistic trajectories described by equations with time-varying coefficients. His theory is based upon the method of adjoint functions, first introduced by G. A. Bliss (3) during World War I. The purpose of the present paper is to make Drenick's theory more complete and definite and to apply it to the problem of automatic navigation of a long-range winged rocket vehicle. A system of control involving a fast computer is suggested, whereby the range errors due to changing atmospheric conditions and deviation from the standard weight of the vehicle are automatically corrected. The main objective here is not, however, to give the final design of such an automatic navigation system, but rather to show the power of the ballistic disturbance theory for solving such problems and the various elements necessary for such a navigational system.

Equations of Motion

In order not to complicate matters, the vehicle is assumed to move in the equatorial plane of the rotating earth (Fig. 1). The planar motion is possible due to the absence of cross Coriolis force in the equatorial plane. The co-ordinate system is fixed with respect to the rotating earth, i.e., actually it rotates with the angular velocity Ω, the speed of earth rotation. The value of Ω is as follows:

$$\Omega = 7.2921 \times 10^{-5} \text{ rad/sec} \quad [1]$$

In the equatorial plane, the position of the vehicle at any time instant t is specified by the radius r and angle θ from the starting point of the vehicle. r_0 is the mean earth radius, its value is

$$r_0 = 20.88 \times 10^6 \text{ ft} \quad [2]$$

If g is the gravitational constant at the surface of the earth without the centrifugal force due to rotation, then

$$g = 32.2577 \text{ ft/sec}^2 \quad [3]$$

Let R and Θ be the force per unit mass acting on the vehicle in the radial and the circumferential directions, respectively. Then the equations of motion of the center of gravity of the vehicle are

$$\left.\begin{aligned} \frac{dr}{dt} &= \dot{r} \\ \frac{d\theta}{dt} &= \dot{\theta} \\ \frac{dr}{dt} &= R + r(\dot{\theta} \pm \Omega)^2 - g\left(\frac{r_0}{r}\right)^2 \\ r\frac{d\theta}{dt} &= \Theta - 2\dot{r}(\dot{\theta} \pm \Omega) \end{aligned}\right\} \quad [4]$$

Received April 28, 1952.

[1] Robert H. Goddard Professor of Jet Propulsion.

[2] Daniel and Florence Guggenheim Jet Propulsion Fellows. The computations involved in this paper were carried out by R. C. Evans, Daniel and Florence Guggenheim Jet Propulsion Fellow, and F. W. Hartwig, Captain U.S.A.F., in addition to the junior authors.

[3] Numbers in parentheses refer to the References on page 199.

where the plus sign in the second terms to the right will be valid for flights toward the east, and the minus sign for flights toward the west.

The forces acting on the center of gravity of the vehicle are the thrust f, the lift L, and the drag D (Fig. 1). Let W be the instantaneous weight of the vehicle with respect to g, and V the magnitude of air velocity relative to the vehicle. Then it is convenient to introduce the parameters Ψ, Λ, and Δ as follows:

$$\Psi = \frac{fg}{W}, \qquad \Lambda = \frac{Lg}{WV}, \qquad \Delta = \frac{Dg}{WV} \quad \ldots\ldots [5]$$

It will be assumed that the natural wind velocity w is in the horizontal direction, positive if it is a head wind. w is considered as a function of altitude r. If v_r is the radial velocity and v_θ the circumferential velocity, i.e.,

$$\left.\begin{aligned} v_r &= \dot{r} \\ v_\theta &= r\dot{\theta} \end{aligned}\right\} \quad \ldots\ldots [6]$$

then relative air velocity V is computed as

$$V^2 = v_r^2 + (v_\theta + w)^2 \quad \ldots\ldots [7]$$

If β is the angle between the thrust line and the horizontal direction, then the components of forces R and Θ per unit mass are

$$\left.\begin{aligned} R &= \Psi \sin\beta + (v_\theta + w)\Lambda - v_r\Delta \\ \Theta &= \Psi \cos\beta - v_r\Lambda - (v_\theta + w)\Delta \end{aligned}\right\} \quad \ldots\ldots [8]$$

If N is the moment of forces about the center of gravity, divided by the moment of inertia of the vehicle, the equation for the angular acceleration is

$$\frac{d\dot{\beta}}{dt} = \frac{d\dot{\theta}}{dt} + N \quad \ldots\ldots [9]$$

To completely specify the motion of the vehicle, the lift L, the drag D, and the moment m about the center of gravity have to be given as functions of time. According to the aerodynamic convention, the L and D will be expressed in terms of the lift coefficient C_L and the drag coefficient C_D as follows:

$$\left.\begin{aligned} L &= \frac{1}{2}\rho V^2 S C_L \\ D &= \frac{1}{2}\rho V^2 S C_D \end{aligned}\right\} \quad \ldots\ldots [10]$$

where ρ is the air density, a function of the altitude r, and S is a fixed reference area, say the wing area of the vehicle. In the present problem, since the motion of the vehicle is restricted to the equatorial plane, the attitude of the vehicle essential for aerodynamic calculations is determined by the angle of attack[4] α, i.e., the angle between the thrust line, or body axis and the relative air velocity vector (Fig. 1). The control on the motion of the vehicle is affected, however, through the elevator angle ϵ. The parameters which will affect C_L and C_D are thus α and ϵ. In addition, the aerodynamic coefficients are functions of the Reynolds number Re and the Mach number M. Thus

$$\left.\begin{aligned} C_L &= C_L(\alpha, \epsilon, M, Re) \\ C_D &= C_D(\alpha, \epsilon, M, Re) \end{aligned}\right\} \quad \ldots\ldots [11]$$

[4] Drenick (Ref. 2) seems to take the sense of the angle of attack α opposite to the convention of aerodynamicists.

It will be assumed that the thrust line passes through the center of gravity of the vehicle; thus the thrust gives no moment. Since the angular motion of the vehicle during the powered flight is expected to be slow, the jet damping moment of the rocket is negligible. The only moment acting on the vehicle is then the aerodynamic moment m. m can be also expressed as a coefficient C_M as follows:

$$m = \frac{1}{2}\rho V^2 S l C_M \quad \ldots\ldots [12]$$

where l is a reference length. The moment coefficient C_M is again a function of the four parameters α, ϵ, M, and Re, or

$$C_M = C_M(\alpha, \epsilon, M, Re) \quad \ldots\ldots [13]$$

If I is the moment of inertia of the vehicle, then the magnitude of N in Equation [9] is

$$N = m/I \quad \ldots\ldots [14]$$

With the rotations defined above, the system of equations of motion is as follows:

$$\left.\begin{aligned} \frac{dr}{dt} &= v_r \\ \frac{d\theta}{dt} &= v_\theta / r \\ \frac{d\beta}{dt} &= \dot{\beta} \\ \frac{dv_r}{dt} &= \Psi \sin\beta + (v_\theta + w)\Lambda - v_r\Delta + r\left(\frac{v_\theta}{r} \pm \Omega\right)^2 - g\left(\frac{r_0}{r}\right)^2 = F \\ \frac{dv_\theta}{dt} &= \Psi \cos\beta - v_r\Lambda - (v_\theta + w)\Delta - 2v_r\left(\frac{v_\theta}{r} + \Omega\right) + \frac{v_\theta v_r}{r} = G \\ \frac{d\dot{\beta}}{dt} &= \frac{1}{r}\{\Psi \cos\beta - v_r\Lambda - (v_\theta + w)\Delta\} - 2\frac{v_r}{r}\left(\frac{v_\theta}{r} + \Omega\right) + N = H \end{aligned}\right\} \quad \ldots\ldots [15]$$

This system of equations is a set of first order equations for the six unknowns r, θ, β, v_r, v_θ, and $\dot{\beta}$. To solve it,

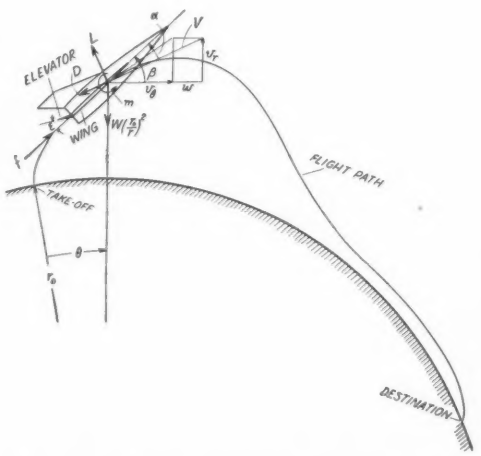

FIG. 1 FLIGHT PATH OF A LONG RANGE WINGED ROCKET
(The size of the vehicle is greatly magnified for clear graphical representation.)

the six initial values at the start, $t = 0$, for the unknowns must be specified. In addition, the thrust f, the weight W, the moment of inertia I must be given for every time instant. To determine the aerodynamic forces, the elevator angle ϵ must be specified as a function of time. The properties of the atmosphere must be known; i.e., wind velocity w, air density ρ, air viscosity and air velocity of sound must be given as functions of the altitude r. The angle of attack α of the vehicle cannot be specified; it is a quantity to be computed from the angle β and the relative air velocity vector V.

Normal Flight Path

Let the properties of the atmosphere be standardized and known as the properties of the normal atmosphere. The average characteristics of the vehicle and its power plant can be taken to be representative. Then if the elevator angle ϵ is given as a function of time, the flight path of the vehicle is determined and can be calculated by integrating the system of Equation [15]. The actual execution of this computation will be probably done on an electro-mechanical computer. This flight path of a standardized vehicle in normal atmosphere can be called the normal flight path.

The dominating feature of the normal flight path is its range. This range is the distance between the take-off point and the landing point. The problem of navigation is then to calculate the proper time for cut-off of the rocket and the proper variation of the elevator angle during flight so that the range is that desired. This problem of navigation for the standardized vehicle in normal atmosphere can be solved mathematically before the actual take-off of the vehicle, since all information for the normal flight path is known or specified beforehand.

Disturbance Equations

Natural atmospheric characteristics do not, of course, coincide with those assumed for the atmosphere. The wind velocity at each altitude changes according to the weather conditions; the temperature T is also a varying quantity. Therefore one should expect variations from the normal flight path due to changes in atmospheric conditions. The actual vehicle also may be somewhat different from the standardized vehicle, in weight, rocket performance, etc. Therefore actual flight path will be different from the normal flight path if the same elevator angle programming is used. The problem of navigation of an actual vehicle is that of correcting the elevator angle programming so that the range of the actual flight will be the same as the normal flight path and the destination is reached without error. Due to the rapidity of flight, this navigational problem cannot be solved by conventional method; but should be solved by an automatic computing system, which responds to every deviation from the normal conditions with a speed approaching instant action.

The general problem of automatic navigation is very difficult indeed. However, the deviations from the normal conditions are expected to be small, since the normal flight path is, after all, a good representation of the average situation. This fact immediately suggests that only first order quantities in deviations need be considered. This "linearization" is the basis of the ballistic disturbance theory and the present theory of automatic navigation.

Let quantities of the normal flight path be denoted by a bar over them and deviations by the δ sign. Thus for the actual flight path,

$$\left.\begin{aligned} r &= \bar{r} + \delta r, \qquad \theta = \bar{\theta} + \delta\theta, \qquad \beta = \bar{\beta} + \delta\beta \\ v_r &= \bar{v}_r + \delta v_r, \qquad v_\theta = \bar{v}_\theta + \delta v_\theta, \qquad \dot{\beta} = \bar{\dot{\beta}} + \delta\dot{\beta} \end{aligned}\right\} \quad \ldots [16]$$

The deviations of the actual atmosphere from the normal atmosphere are expressed as the deviation of density $\delta\rho$, the deviation of temperature δT, and the deviation of wind velocity δw; thus

$$\rho = \bar{\rho} + \delta\rho, \qquad T = \bar{T} + \delta T, \qquad w = \bar{w} + \delta w \quad [17]$$

The deviation of the actual vehicle from the normal vehicle is assumed to be limited only to deviation of weight δW and the moment of inertia δI. That is

$$W = \bar{W} + \delta W, \qquad I = \bar{I} + \delta I \ldots\ldots\ldots\ldots [18]$$

The rate of propellant flow and the effective exhaust velocity of the rocket is assumed to be standard. The wing area S and the aerodynamic characteristics of the vehicle as expressed by Equations [11] and [13] are also assumed to be invariant.

By substituting Equations [16], [17], and [18] into the equations of motion, Equation [15], and retaining only first term deviations, one has

$$\left.\begin{aligned} \frac{d\delta r}{dt} &= +\delta v_r \\ \frac{d\delta\theta}{dt} &= -\frac{\bar{v}_\theta}{\bar{r}^2}\delta r + \frac{1}{\bar{r}}\delta v_\theta \\ \frac{d\delta\beta}{dt} &= +\delta\dot{\beta} \end{aligned}\right\} \quad \ldots [19]$$

$$\left.\begin{aligned} \frac{d\delta v_r}{dt} &= a_1\delta r + a_2\delta\beta + a_3\delta v_r + a_4\delta v_\theta + a_5\delta\epsilon + a_6\delta\rho + a_7\delta T + a_8\delta w + a_9\delta W \\ \frac{d\delta v_\theta}{dt} &= b_1\delta r + b_2\delta B\beta + b_3\delta v_r + b_4\delta v_\theta + b_5\delta\epsilon + b_6\delta\rho + b_7\delta T + b_8\delta w + b_9\delta W \\ \frac{d\delta\dot{\beta}}{dt} &= c_1\delta r + c_2\delta\beta + c_3\delta v_r + c_4\delta v_\theta + c_5\delta\epsilon + c_6\delta\rho + c_7\delta T + c_8\delta w + c_9\delta W + c_{10}\delta I \end{aligned}\right\} \quad \ldots [20]$$

The coefficients a's, b's, and c's are partial derivatives of F, G, and H defined by Equations [15], evaluated on the normal flight path. That is, for example,

$$\left.\begin{aligned} a_1 &= \left(\frac{\overline{\partial F}}{\partial r}\right), \quad a_2 = \left(\frac{\overline{\partial F}}{\partial \beta}\right), \quad a_3 = \left(\frac{\overline{\partial F}}{\partial v_r}\right), \\ a_4 &= \left(\frac{\overline{\partial F}}{\partial v_\theta}\right), \quad a_5 = \left(\frac{\overline{\partial F}}{\partial \epsilon}\right), \quad a_6 = \left(\frac{\overline{\partial F}}{\partial \rho}\right), \\ a_7 &= \left(\frac{\overline{\partial F}}{\partial T}\right), \quad a_8 = \left(\frac{\overline{\partial F}}{\partial w}\right), \quad a_9 = \left(\frac{\overline{\partial F}}{\partial W}\right) \end{aligned}\right\} \quad \ldots [21]$$

These coefficients are calculated in detail and given in the Appendix.

The system of Equations [19] and [20] is the system of disturbance equations. They are linear. The coefficients when evaluated on the normal flight path are

finally specified functions of time. If the deviations of the atmospheric properties $\delta\rho$, δT, and δw are known, and if $\delta\epsilon$, δW, and δI are specified, then this system of differential equations determines the deviations δr, $\delta\theta$, $\delta\beta$, δv_r, δv_θ, and $\delta\dot{\beta}$ from the normal trajectory. This is the direct problem in the ballistic disturbance theory. The problem of automatic navigation is however different from this. What is required is the function $\delta\epsilon$, correction to the elevator angle, such that the range error is zero. As suggested by Drenick (2), this navigational problem can best be solved by the method of adjoint functions of Bliss.

Adjoint Functions for Range Correction

The principle of the method of adjoint functions is as follows (3): Let $y_i(t)$, $i = 1, \ldots, n$, be determined by a system of linear equations

$$\frac{dy_i}{dt} = \sum_{j=1}^{n} a_{ij}y_j + Y_i \quad \ldots\ldots[22]$$

where a_{ij} are given coefficients which may be functions of the time t, and Y_i are specified functions of time. Now introduce a new set of functions $\lambda_i(t)$, called the adjoint functions to $y_i(t)$, which satisfy the following system of differential equations

$$\frac{d\lambda_i}{dt} = -\sum_{j=1}^{n} a_{ji}\lambda_j \quad \ldots\ldots[23]$$

By multiplying Equation [22] by λ_i and Equation [23] by y_i, and then adding the resultant equations, it can be shown that

$$\frac{d}{dt}\sum_{i=1}^{n}\lambda_i y_i = \sum_{i=1}^{n}\lambda_i Y_i \quad \ldots\ldots[24]$$

Equation [24] can be integrated from $t = t_1$ to $t = t_2$,

$$\sum_{i=1}^{n}\lambda_i y_i\Big|_{t=t_2} = \sum_{i=1}^{n}\lambda_i y_i\Big|_{t=t_1} + \int_{t=t_1}^{t=t_2}\left(\sum_{i=1}^{n}\lambda_i Y_i\right)dt \quad [25]$$

Bliss named Equation [25] the "Fundamental Formula."

For the present problem, the y_i are the disturbance quantities, i.e.,

$$\left.\begin{array}{lll} y_1 = \delta r, & y_2 = \delta\theta, & y_3 = \delta\beta \\ y_4 = \delta v_r, & y_5 = \delta v_\theta, & y_6 = \delta\dot{\beta} \end{array}\right\} \quad \ldots\ldots[26]$$

Then, according to Equation [20], the adjoint functions λ_i satisfy the following differential equations

$$\left.\begin{array}{rcl} -\dfrac{d\lambda_1}{dt} &=& -\dfrac{\bar{v}_\theta}{\bar{r}^2}\lambda_2 + a_1\lambda_4 + b_1\lambda_5 + c_1\lambda_6 \\ -\dfrac{d\lambda_2}{dt} &=& 0 \\ -\dfrac{d\lambda_3}{dt} &=& a_2\lambda_4 + b_2\lambda_5 + c_2\lambda_6 \\ -\dfrac{d\lambda_4}{dt} &=& \lambda_1 + a_3\lambda_4 + b_3\lambda_5 + c_3\lambda_6 \\ -\dfrac{d\lambda_5}{dt} &=& \dfrac{1}{\bar{r}}\lambda_2 + a_4\lambda_4 + b_4\lambda_5 + c_4\lambda_6 \\ -\dfrac{d\lambda_6}{dt} &=& \lambda_3 \end{array}\right\} \quad \ldots[27]$$

The Y_i are then

$$\left.\begin{array}{l} Y_1 = Y_2 = Y_3 = 0 \\ Y_4 = a_5\delta\epsilon + a_6\delta\rho + a_7\delta T + a_8\delta w + a_9\delta W \\ Y_5 = b_5\delta\epsilon + b_6\delta\rho + b_7\delta T + b_8\delta w + b_9\delta W \\ Y_6 = c_5\delta\epsilon + c_6\delta\rho + c_7\delta T + c_8\delta w + c_9\delta W + c_{10}\delta I \end{array}\right\} \quad \ldots[28]$$

The Equations [27] do not determine the λ-functions completely. To do that, a set of values for λ must be specified at a certain instant. For the problem of automatic navigation, the requirement is vanishing range error; then the required boundary conditions for Equations [27] can be determined as follows: If t_2 is the time instant of landing of the actual vehicle, $\bar{t}_2$ the time instant of landing of the normal flight path, then

$$t_2 = \bar{t}_2 + \delta t_2 \quad \ldots\ldots[29]$$

Similarly with the subscript 2 denoting the quantities at the landing instant

$$\left.\begin{array}{l} r_2 = \bar{r}_2 + \delta r_2 \\ \theta_2 = \bar{\theta}_2 + \delta\theta_2 \end{array}\right\} \quad \ldots\ldots[30]$$

But

$$\left.\begin{array}{l} \delta r_2 = (\bar{v}_r)_{t=\bar{t}_2}\,\delta t_2 + (\delta r)_{t=\bar{t}_2} \\ \delta\theta_2 = \dfrac{1}{r_0}(\bar{v}_\theta)_{t=\bar{t}_2}\,\delta t_2 + (\delta\theta)_{t=\bar{t}_2} \end{array}\right\} \quad \ldots\ldots[31]$$

δr_2 is, however, zero by definition, because landing means contact with the surface of earth, $\bar{r}_2 = r_2 = r_0$. By eliminating δt_2 from the Equations [31],

$$\delta\theta_2 = \left[-\frac{1}{\bar{r}}\left(\frac{\bar{v}_\theta}{\bar{v}_r}\right)\delta r + \delta\theta\right]_{t=\bar{t}_2} \quad \ldots\ldots[32]$$

Therefore if the magnitudes of λ_i at the landing instant $t = \bar{t}_2$ are specified as

$$\left.\begin{array}{l} \lambda_1 = -\dfrac{1}{\bar{r}}\left(\dfrac{\bar{v}_\theta}{\bar{v}_r}\right) \\ \lambda_2 = 1 \\ \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0 \end{array}\right\} t = \bar{t}_2 \quad \ldots\ldots[33]$$

then the error in range is given by

$$\delta\theta_2 = \sum_{i=1}^{n}\lambda_i y_i\Big|_{t=\bar{t}_2} = [\lambda_1\delta r + \lambda_2\delta\theta + \lambda_3\delta\beta + \lambda_4\delta v_r + \lambda_5\delta v_\theta + \lambda_6\delta\dot{\beta}]_{t=\bar{t}_2} \quad [34]$$

When the normal flight path is determined, the coefficients in Equation [27] are specified as functions of time. These equations together with the end conditions of Equation [33] then determine the adjoint functions λ_i. The integration has to be performed "backwards" for $t < \bar{t}_2$, by perhaps an electro-mechanical computer. With the adjoint functions so determined, one can use the Fundamental Formula of Equation [25] to modify the equation for the range error given by Equation [34]: Let $\bar{t}_1$ denote the time instant for the power cut-off. Then the condition for the error $\delta\theta_2$ in range to be zero can be expressed as

$$\left.\begin{array}{l} \delta\theta_2 = 0 = [\lambda_1\delta r + \lambda_2\delta\theta + \lambda_3\delta\beta + \lambda_4\delta v_r + \lambda_5\delta v_\theta + \lambda_6\delta\dot{\beta}]_{t=\bar{t}_1} \\ \quad + \displaystyle\int_{\bar{t}_1}^{\bar{t}_2}[\lambda_4 Y_4 + \lambda_5 Y_5 + \lambda_6 Y_6]\,dt \end{array}\right\} \quad [35]$$

This is the basic equation for automatic navigation. It will be exploited presently.

Cut-Off Condition

The condition of Equation [35] for arbitrary dis-

turbances can be broken down into two parts: The sum and the integral to be set to zero separately. Therefore the condition to be satisfied at the normal cut-off instant $\bar{t}_1$ is

$$[\lambda_1\delta r + \lambda_2\delta\theta + \lambda_3\delta\beta + \lambda_4\delta v_r + \lambda_5\delta v_\theta + \lambda_6\delta\dot{\beta}]_{t=\bar{t}_1} = 0 \quad [36]$$

Since the normal cut-off instant $\bar{t}_1$ is a standard time instant, but not necessarily the actual cut-off instant t_1, i.e.,

$$t_1 = \bar{t}_1 + \delta t_1 \quad [37]$$

Equation [36] should be converted into a more useful form involving the quantities at the actual cut-off instant. This is easily done, because up to the first order quantities, according to Equation [16]

$$(\delta r)_{t=\bar{t}_1} = (r)_{t=t_1} - \left(\frac{\bar{dr}}{dt}\right)_{t=\bar{t}_1}\delta t_1 - (\bar{r})_{t=\bar{t}_1}$$

Or

$$(\delta r)_{t=\bar{t}_1} = (r)_{t=t_1} - (\bar{r})_{t=\bar{t}_1} - (\bar{v}_r)_{t=\bar{t}_1}\,\delta t_1$$

Similarly,

$$(\delta\theta)_{t=\bar{t}_1} = (\theta)_{t=t_1} - (\bar{\theta})_{t=\bar{t}_1} - \left(\frac{1}{\bar{r}}\,\bar{v}_\theta\right)_{t=\bar{t}_1}\delta t_1$$

$$(\delta\beta)_{t=\bar{t}_1} = (\beta)_{t=t_1} - (\bar{\beta})_{t=\bar{t}_1} - (\dot{\beta})_{t=\bar{t}_1}\,\delta t_1$$

$$(\delta v_r)_{t=\bar{t}_1} = (v_r)_{t=t_1} - (\bar{v}_r)_{t=\bar{t}_1} - (\bar{F})_{t=\bar{t}_1}\,\delta t_1$$

$$(\delta v_\theta)_{t=\bar{t}_1} = (v_\theta)_{t=t_1} - (\bar{v}_\theta)_{t=\bar{t}_1} - (\bar{G})_{t=\bar{t}_1}\,\delta t_1$$

$$(\delta\dot{\beta})_{t=\bar{t}_1} = (\dot{\beta})_{t=t_1} - (\bar{\dot{\beta}})_{t=\bar{t}_1} - (\bar{H})_{t=\bar{t}_1}\,\delta t_1$$

where $\bar{F}$, $\bar{G}$, and $\bar{H}$, are the values of these quantities given by Equation [15] evaluated on the normal flight path. In fact, they should be evaluated at an instant just before the normal cut-off time $\bar{t}_1$ so that the accelerating force of the rocket is included and the rates of change of velocities are those of a powered flight. Now define J and $\bar{J}$ as follows,

$$J = [\lambda_1^* r + \lambda_2^*\theta + \lambda_3^*\beta + \lambda_4^* v_r + \lambda_5^* v_\theta + \lambda_6^*\dot{\beta}]_{t=t_1} \quad [38]$$

and

$$\bar{J} = [\lambda_1^*\bar{r} + \lambda_2^*\bar{\theta} + \lambda_3^*\bar{\beta} + \lambda_4^*\bar{v}_r + \lambda_5^* v_\theta + \lambda_6^*\bar{\dot{\beta}}]_{t=\bar{t}_1}$$

where λ_i^* are the values of λ_i evaluated at the normal cut-off time t_1. Then the condition to be satisfied at the actual cut-off instant t_1 is

$$J = \bar{J} + \left\{\lambda_1^*\bar{v}_r + \lambda_2^*\frac{\bar{v}_\theta}{\bar{r}} + \lambda_3^*\bar{\beta} + \lambda_4^*\bar{F} + \lambda_5^*\bar{G} + \lambda_6^*\bar{H}\right\}_{t=t_1}(t_1 - \bar{t}_1) \quad [39]$$

This is the equation to determine the proper instant of power cut-off.

When the normal flight path is known, $\bar{J}$ and the quantity within the bracket to the right of Equation [39] are fixed. Then the whole right-hand side of Equation [39] can be considered as a linearly increasing function of time t, if t is substituted for t_1. Simultaneously J can be computed at every instant before cut-off by using the predetermined λ_i^* and values of position and velocity of the actual vehicle obtained by tracking stations. The magnitudes of the quantities on the two sides of Equation [39] can then be continuously compared. When they are equal to each other, condition [39] is satisfied. Then the power cut-off signal is given and the rocket power is shut off.[5]

Condition for Automatic Navigation

When the rocket power is shut off earlier or later than the normal cut-off instant $\bar{t}_1$, the propellant left in the tank, if not dumped, will alter the weight W and the moment of inertia I of the vehicle. It is also possible that the pay load of the vehicle is not that specified for the standard vehicle. Then after power-off, there is a fixed δW and δI, fixed in the sense that they do not change with time, and are known once the power cut-off is affected. Of different character are the deviations $\delta\rho$, δT, δw of the actual atmosphere from the standard atmosphere. These are not known unless they are measured. In the following, it is proposed to use the vehicle itself as a measuring instrument, and proceed as follows:

After the cut-off condition is satisfied, the condition for zero range error is that the integral in Equation [35] should vanish. Now since the Y_i in that integrand involving arbitrary disturbances $\delta\rho$, δT, and δw is not known beforehand, this condition can be satisfied only if the integrand itself vanishes. That is, according to Equations [28],

$$(\lambda_4 a_5 + \lambda_5 b_5 + \lambda_6 c_5)\delta\epsilon + (\lambda_4 a_6 + \lambda_5 b_6 + \lambda_6 c_6)\delta\rho + (\lambda_4 a_7 + \lambda_5 b_7 + \lambda_6 c_7)\delta T + (\lambda_4 a_8 + \lambda_5 b_8 + \lambda_6 c_8)\delta w + (\lambda_4 a_9 + \lambda_5 b_9 + \lambda_6 c_9)\delta W + \lambda_6 c_{10}\delta I = 0$$

Or, with the following notation

$$\left.\begin{aligned} d_5 &= \lambda_4 a_5 + \lambda_5 b_5 + \lambda_6 c_5 \\ d_6 &= \lambda_4 a_6 + \lambda_5 b_6 + \lambda_6 c_6 \\ d_7 &= \lambda_4 a_7 + \lambda_5 b_7 + \lambda_6 c_7 \\ d_8 &= \lambda_4 a_8 + \lambda_5 b_8 + \lambda_6 c_8 \\ D &= -(\lambda_4 a_9 + \lambda_5 b_9 + \lambda_6 c_9)\delta W - \lambda_6 c_{10}\delta I \end{aligned}\right\} \quad [40]$$

this condition can be written as

$$d_5\delta\epsilon + d_6\delta\rho + d_7\delta T + d_8\delta w = D \quad [41]$$

Equation [20] can be rewritten as

$$\left.\begin{aligned} a_5\delta\epsilon + a_6\delta\rho + a_7\delta T + a_8\delta w &= A \\ b_5\delta\epsilon + b_6\delta\rho + b_7\delta T + b_8\delta w &= B \\ c_5\delta\epsilon + c_6\delta\rho + c_7\delta T + c_8\delta w &= C \end{aligned}\right\} \quad [42]$$

where

$$\left.\begin{aligned} A &= \frac{d\delta v_r}{dt} - a_1\delta r - a_2\delta\beta - a_3\delta v_r - a_4\delta v_\theta - a_7\delta W \\ B &= \frac{d\delta v_\theta}{dt} - b_1\delta r - b_2\delta\beta - b_3\delta v_r - b_4\delta v_\theta - b_7\delta W \\ C &= \frac{d\delta\dot{\beta}}{dt} - c_1\delta r - c_2\delta\beta - c_3\delta v_r - c_4\delta v_\theta - c_9\delta W - c_{10}\delta I \end{aligned}\right\} \quad [43]$$

If the tracking stations for the vehicle will measure the quantities A, B, and C, then the atmospheric disturbances $\delta\rho$, δT, and δw can be determined by solving for these variations using Equation [42]. This is essentially using the vehicle itself as a measuring instrument for $\delta\rho$, δT, and δw. When $\delta\rho$, δT, and δw are known, Equation [41] gives the proper elevator angle correction $\delta\epsilon$.

A mathematically equivalent way to calculate $\delta\epsilon$

[5] Drenick's cut-off condition (Ref. 2) differs from that of Equation [39] in that the second term to the right is not present. It seems that this neglect is not justified and will introduce first order range error.

would be to directly solve for $\delta\epsilon$ using the system of Equations [41] and [42]. Thus

$$\delta\epsilon = \frac{\begin{vmatrix} a_5 & a_6 & a_7 & a_8 \\ b_5 & b_6 & b_7 & b_8 \\ c_5 & c_6 & c_7 & c_8 \\ d_5 & d_6 & d_7 & d_8 \end{vmatrix}}{\begin{vmatrix} A & a_6 & a_7 & a_8 \\ B & b_6 & b_7 & b_8 \\ C & c_6 & c_7 & c_8 \\ D & d_6 & d_7 & d_8 \end{vmatrix}} \quad \ldots\ldots[44]$$

This equation specifies the necessary change in the elevator angle at every instant to be calculated from the quantities a's, b's, c's, and A, B, C, D at the same instant. These quantities consist partly of predetermined information from the normal flight path, and partly of measured information on the position and the velocities of vehicle obtained by tracking the vehicle. At high altitudes where the air density is very small, the aerodynamic forces will be almost negligible in comparison with the gravitational and inertia forces. Then the quantities A, B, and C of Equation [43] will be the small difference of large magnitudes. These are then the quantities most difficult to determine accurately. If the actual elevator angle is made to conform with the one calculated by Equation [44], then in conjunction with the proper power cut-off as specified in the last section the vehicle will be navigated to the chosen landing point in spite of the atmospheric disturbances.

Discussion

When the general character of the flight path is chosen from the over-all engineering considerations, the first step is the calculation of the normal flight path using the properties of the standard atmosphere and the expected performance of the vehicle with normal weight. The knowledge of the normal flight path then determines the a's, b's, and c's. The Equation [27] together with the end conditions of Equation [33] allows the calculation of the adjoint functions λ_i. All this information should be on hand before the actual flight of the vehicle, and may be called the "stored data."

Before the power cut-off, the elevator angle may be programmed according to that for the normal flight path, and the stability of the vehicle is supplied by the jet vanes or by the auxiliary rockets. The tracking stations, however, go immediately into action and supply the vehicle with information on its positions and velocities. This information goes first into the cut-off computer which, using the stored information, continuously compares the magnitudes of quantities on the two sides of Equation [39], the cut-off condition. When that condition is satisfied, the power cut-off is affected.

At the instant of power cut-off, the tracking information is switched to the computer for automatic navigation. The instant of power cut-off also fixes the amount of propellant in the tank and thus determines the variations of weight δW and inertia moment δI from the standard. This information together with the stored data on the normal flight path then allows the computer to generate the elevator correction angle $\delta\epsilon$ according to Equations [40], [43], and [44]. Theoretically the value of $\delta\epsilon$ must be obtained without time delay from the instant when the information is received, because Equation [44] is a condition of equality of two quantities evaluated at identical time instants. Practically there will be time delay due to the finite computing time. However, it is now clear that this computing time must be made very short in order to satisfy the condition of automatic navigation as accurately as possible. The computed correction $\delta\epsilon$ combined with the elevator angle $\bar{\epsilon}$ determined for the normal flight path then gives the actual elevator angle setting ϵ. The design of the control mechanism for the elevator from here on follows the conventional feed-back servomechanism, with the usual criteria of quick action, stability, and accuracy. The general scheme of the automatic navigation of the vehicle then can be represented by the sketch in Fig. 2.

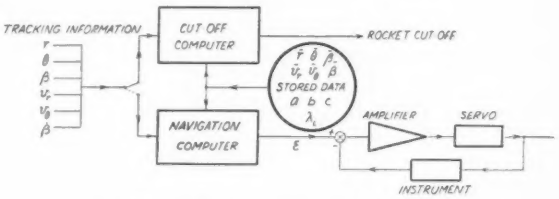

FIG. 2. SYSTEM OF AUTOMATIC NAVIGATION

The primary purpose or the "duty" of the computer is then to properly digest the dynamic and the aerodynamic information of the vehicle and thus to decide the right flight path correction that will insure landing at the chosen destination. The purpose or the duty of the elevator servo is now simply to follow the command of the computer. If the servo is considered to be *internal* to the vehicle, then the computer can be considered to be a mechanism to account for the *external* conditions of the vehicle. This separation of the *internal* from the *external* is not necessary for the control of conventional aircraft. For conventional aircraft, the basic disturbance dynamic equations have constant coefficients and the computer can be merged with the servo through a single "amplifier" of essentially RC-circuits.

The computers envisaged here are carried in the vehicle and receive the information on positions and velocities from the fixed ground tracking stations (4). It will be beyond the scope of the present paper to discuss their design. But the required accuracy and speed to generate proper signal almost instantly would indicate that they should be of the electronic digital type. If this is indeed the proper type to be used, then the separation of cut-off computer and the navigation computer is not necessary. All could be done in one computer by proper programming.

Appendix

Calculation of Coefficients

The quantities F, G, and H are defined by Equation [15]. They contain the parameters Ψ, Λ, Δ, and N.

According to their definition as given by Equations [5] and [14], they can be written as follows:

$$\left.\begin{aligned}\Psi &= \frac{fg}{W}\\ \Lambda &= \frac{g}{W}\frac{1}{2}\rho S C_L\sqrt{v_r^2+(v_\theta+w)^2}\\ \Delta &= \frac{g}{W}\frac{1}{2}\rho S C_D\sqrt{v_r^2+(v_\theta+w)^2}\\ N &= \frac{1}{I}\frac{1}{2}\rho S C_M\{v_r^2+(v_\theta+w)^2\}\end{aligned}\right\}\quad\ldots\ldots[45]$$

where the aerodynamic coefficients C_L, C_D, C_M are functions of angle of attack α, elevator angle ϵ, Mach number M, and the Reynolds number Re. These aerodynamic parameters are related to quantities immediately connected with the flight path as follows:

$$\alpha = \beta - \tan^{-1}\frac{v_r}{v_\theta+w},\quad M = V/a(r),\quad Re = \rho V l/\mu(r)\qquad\ldots[46]$$

where $a(r)$ is the sound velocity in atmosphere, and $\mu(r)$ is the coefficient of viscosity of air, both functions of altitude r. In the following calculation, the thrust f will be considered to be a function of altitude only. It is also assumed that the composition of atmosphere at different altitudes remains the same as that of the standard atmosphere, only the density ρ and the temperature T change. Thus the variations of a and μ at any altitude are variations due to temperature T.

For Ψ

$$\frac{\partial\Psi}{\partial r} = \frac{g}{W}\frac{\partial f}{\partial r},\qquad \frac{\partial\Psi}{\partial W} = -\frac{\Psi}{W}\qquad\ldots\ldots[47]$$

All other partial derivatives are zero.

For Λ

$$\left.\begin{aligned}\frac{\partial\Lambda}{\partial r} &= \Lambda\left[\frac{1}{\rho}\frac{d\rho}{dr}\left\{1+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}\right\}+\frac{1}{V^2}\frac{dw}{dr}\times\right.\\ &\quad\left\{\left(\frac{M}{C_L}\frac{\partial C_L}{\partial M}+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}+1\right)\times\right.\\ &\quad\left.\left.(v_\theta+w)+\frac{1}{C_L}\frac{\partial C_L}{\partial\alpha}v_r\right\}-\right.\\ &\quad\left.\frac{M}{C_L}\frac{\partial C_L}{\partial M}\frac{1}{a}\frac{da}{dr}-\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}\frac{1}{\mu}\frac{d\mu}{dr}\right]\\ \frac{\partial\Lambda}{\partial v_r} &= \Lambda\frac{v_r}{V^2}\left[\frac{M}{C_L}\frac{\partial C_L}{\partial M}+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}+\right.\\ &\quad\left.1-\frac{1}{C_L}\frac{\partial C_L}{\partial\alpha}\frac{v_\theta+w}{v_r}\right]\\ \frac{\partial\Lambda}{\partial v_\theta} &= \Lambda\frac{v_\theta+w}{V^2}\left[\frac{M}{C_L}\frac{\partial C_L}{\partial M}+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}+\right.\\ &\quad\left.1+\frac{1}{C_L}\frac{\partial C_L}{\partial\alpha}\frac{v_r}{v_\theta+w}\right]\\ \frac{\partial\Lambda}{\partial\beta} &= \Lambda\frac{1}{C_L}\frac{\partial C_L}{\partial\alpha}\\ \frac{\partial\Lambda}{\partial\epsilon} &= \Lambda\frac{1}{C_L}\frac{\partial C_L}{\partial\epsilon}\\ \frac{\partial\Lambda}{\partial\rho} &= \Lambda\frac{1}{\rho}\left[1+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}\right]\\ \frac{\partial\Lambda}{\partial T} &= -\Lambda\left[\frac{M}{C_L}\frac{\partial C_L}{\partial M}\frac{1}{2T}+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}\frac{1}{\mu}\frac{\partial\mu}{\partial T}\right]\\ \frac{\partial\Lambda}{\partial w} &= \Lambda\frac{v_\theta+w}{V^2}\left[\frac{M}{C_L}\frac{\partial C_L}{\partial M}+\frac{Re}{C_L}\frac{\partial C_L}{\partial Re}+\right.\\ &\quad\left.1+\frac{1}{C_L}\frac{\partial C_L}{\partial\alpha}\frac{v_r}{v_\theta+w}\right]=\frac{\partial\Delta}{\partial v_\theta}\\ \frac{\partial\Lambda}{\Lambda W} &= -\frac{\Lambda}{W}\end{aligned}\right\}\quad\ldots[48]$$

For Δ, the partial derivatives are obtained from the above equations by substituting Δ for Λ, and C_D for C_L.

For N,

$$\left.\begin{aligned}\frac{\partial N}{\partial r} &= N\left[\frac{1}{\rho}\frac{d\rho}{dr}\left\{1+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}\right\}+\frac{1}{V^2}\frac{dw}{dr}\times\right.\\ &\quad\left\{\left(\frac{M}{C_M}\frac{\partial C_M}{\partial M}+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}+2\right)\times\right.\\ &\quad\left.\left.(v_\theta+w)+\frac{1}{C_M}\frac{\partial C_M}{\partial\alpha}v_r\right\}-\right.\\ &\quad\left.\frac{M}{C_M}\frac{\partial C_M}{\partial M}\frac{1}{a}\frac{da}{dr}-\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}\frac{1}{\mu}\frac{d\mu}{dr}\right]\\ \frac{\partial N}{\partial v_r} &= N\frac{v_r}{V^2}\left[\frac{M}{C_M}\frac{\partial C_M}{\partial M}+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}+\right.\\ &\quad\left.2-\frac{1}{C_M}\frac{\partial C_M}{\partial\alpha}\frac{v_\theta+w}{v_r}\right]\\ \frac{\partial N}{\partial v_\theta} &= N\frac{(v_\theta+w)}{V^2}\left[\frac{M}{C_M}\frac{\partial C_M}{\partial M}+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}+\right.\\ &\quad\left.2+\frac{1}{C_M}\frac{\partial C_M}{\partial\alpha}\frac{v_r}{v_\theta+w}\right]\\ \frac{\partial N}{\partial\beta} &= N\frac{1}{C_M}\frac{\partial C_M}{\partial\alpha}\\ \frac{\partial N}{\partial\epsilon} &= N\frac{1}{C_M}\frac{\partial C_M}{\partial\epsilon}\\ \frac{\partial N}{\partial\rho} &= N\frac{1}{\rho}\left[1+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}\right]\\ \frac{\partial N}{\partial T} &= -N\left[\frac{M}{C_M}\frac{\partial C_M}{\partial M}\frac{1}{2T}+\frac{Re}{C_M}\frac{\partial C_M}{\partial Re}\frac{1}{\mu}\frac{\partial\mu}{\partial T}\right]\\ \frac{\partial N}{\partial w} &= \frac{\partial N}{\partial v_\theta}\\ \frac{\partial N}{\partial I} &= -\frac{N}{I}\end{aligned}\right\}\quad\ldots[49]$$

With these partial derivatives, the coefficients a's, b's, and c's can be easily calculated:

$$\left.\begin{aligned}a_1 &= \frac{\partial F}{\partial r} = \frac{\partial\Psi}{\partial r}\sin\beta+\frac{dw}{dr}\Lambda+(v_\theta+w)\frac{\partial\Lambda}{\partial r}-\\ &\quad v_r\frac{\partial\Delta}{\partial r}+\left(\frac{v_\theta}{r}\pm\Omega\right)^2-2\frac{v_\theta}{r}\left(\frac{v_\theta}{r}\pm\Omega\right)+\\ &\quad 2\frac{g}{r}\left(\frac{r_0}{r}\right)^2\\ a_2 &= \frac{\partial F}{\partial\beta} = \Psi\cos\beta+(v_\theta+w)\frac{\partial\Lambda}{\partial\beta}-v_r\frac{\partial\Delta}{\partial\beta}\\ a_3 &= \frac{\partial F}{\partial v_r} = (v_\theta+w)\frac{\partial\Lambda}{\partial v_r}-\Delta-v_r\frac{\partial\Delta}{\partial v_r}\\ a_4 &= \frac{\partial F}{\partial v_\theta} = \Lambda+(v_\theta+w)\frac{\partial\Lambda}{\partial v_\theta}-v_r\frac{\partial\Delta}{\partial v_\theta}+2\left(\frac{v_\theta}{r}\pm\Omega\right)\\ a_5 &= \frac{\partial F}{\partial\epsilon} = (v_\theta+w)\frac{\partial\Lambda}{\partial\epsilon}-v_r\frac{\partial\Delta}{\partial\epsilon}\\ a_6 &= \frac{\partial F}{\partial\rho} = (v_\theta+w)\frac{\partial\Lambda}{\partial\rho}-v_r\frac{\partial\Delta}{\partial\rho}\\ a_7 &= \frac{\partial F}{\partial T} = (v_\theta+w)\frac{\partial\Lambda}{\partial T}-v_r\frac{\partial\Delta}{\partial T}\\ a_8 &= \frac{\partial F}{\partial w} = \Lambda+(v_\theta+w)\frac{\partial\Lambda}{\partial w}-v_r\frac{\partial\Delta}{\partial w}\\ a_9 &= \frac{\partial F}{\partial W} = \frac{\partial\Psi}{\partial W}\sin\beta+(v_\theta+w)\frac{\partial\Lambda}{\partial W}-v_r\frac{\partial\Delta}{\partial W}\end{aligned}\right\}\quad\ldots[50]$$

$$\left.\begin{aligned}b_1 &= \frac{\partial G}{\partial r} = \frac{\partial\Psi}{\partial r}\cos\beta-v_r\frac{\partial\Delta}{\partial r}-\frac{dw}{dr}\Delta-\\ &\quad(v_\theta+w)\frac{\partial\Delta}{\partial r}+\frac{v_r v_\theta}{r^2}\\ b_2 &= \frac{\partial G}{\partial\beta} = -\Psi\sin\beta-v_r\frac{\partial\Delta}{\partial\beta}-(v_\theta+w)\frac{\partial\Delta}{\partial\beta}\end{aligned}\right\}\quad\ldots[51]$$

$$
\left.
\begin{aligned}
b_3 &= \frac{\partial G}{\partial v_r} = -\Lambda - v_r\frac{\partial \Lambda}{\partial v_r} - (v_\theta + w)\frac{\partial \Delta}{\partial v_r} - 2\left(\frac{1}{2}\frac{v_\theta}{r} \pm \Omega\right)\\
b_4 &= \frac{\partial G}{\partial v_\theta} = -v_r\frac{\partial \Lambda}{\partial v_\theta} - \Delta - (v_\theta + w)\frac{\partial \Delta}{\partial v_\theta} - \frac{v_r}{r}\\
b_5 &= \frac{\partial G}{\partial \epsilon} = -v_r\frac{\partial \Lambda}{\partial \epsilon} - (v_\theta + w)\frac{\partial \Delta}{\partial \epsilon}\\
b_6 &= \frac{\partial G}{\partial \rho} = -v_r\frac{\partial \Lambda}{\partial \rho} - (v_\theta + w)\frac{\partial \Delta}{\partial \rho}\\
b_7 &= \frac{\partial G}{\partial T} = -v_r\frac{\partial \Lambda}{\partial T} - (v_\theta + w)\frac{\partial \Delta}{\partial T}\\
b_8 &= \frac{\partial G}{\partial w} = -v_r\frac{\partial \Lambda}{\partial w} - \Delta - (v_\theta + w)\frac{\partial \Delta}{\partial w}\\
b_9 &= \frac{\partial G}{\partial W} = \frac{\partial \Psi}{\partial W}\cos\beta - v_r\frac{\partial \Lambda}{\partial W} - (v_\theta + w)\frac{\partial \Delta}{\partial W}
\end{aligned}
\right\} \ldots[51]\ \text{(cont'd)}
$$

$$
\left.
\begin{aligned}
c_1 &= \frac{\partial H}{\partial r} = -\frac{1}{r^2}\left\{\Psi\cos\beta - v_r\Lambda - (v_\theta + w)\Delta - 2v_r\left(\frac{v_\theta}{r} \pm \Omega\right)\right\} + \frac{1}{r}\left\{\frac{\partial \Psi}{\partial r}\cos\beta - v_r\frac{\partial \Lambda}{\partial r} - (v_\theta + w)\frac{\partial \Delta}{\partial r} - \frac{dw}{dr}\Delta + 2\frac{v_r v_\theta}{r^2}\right\} + \frac{\partial N}{\partial r}\\
c_2 &= \frac{\partial H}{\partial \beta} = \frac{1}{r}\left\{-\Psi\sin\beta - v_r\frac{\partial \Lambda}{\partial \beta} - (v_\theta + w)\frac{\partial \Delta}{\partial \beta}\right\} + \frac{\partial N}{\partial \beta}\\
c_3 &= \frac{\partial H}{\partial v_r} = \frac{1}{r}\left\{-\Lambda - v_r\frac{\partial \Lambda}{\partial v_r} - (v_\theta + w)\frac{\partial \Delta}{\partial v_r} - 2\left(\frac{v_\theta}{r} \pm \Omega\right)\right\} + \frac{\partial N}{\partial v_r}\\
c_4 &= \frac{\partial H}{\partial v_\theta} = \frac{1}{r}\left\{-v_r\frac{\partial \Lambda}{\partial v_\theta} - \Delta - (v_\theta + w)\frac{\partial \Delta}{\partial v_\theta} - 2\frac{v_r}{r}\right\} + \frac{\partial N}{\partial v_\theta}\\
c_5 &= \frac{\partial H}{\partial \epsilon} = \frac{1}{r}\left\{-v_r\frac{\partial \Lambda}{\partial \epsilon} - (v_\theta + w)\frac{\partial \Delta}{\partial \epsilon}\right\} + \frac{\partial N}{\partial \epsilon}\\
c_6 &= \frac{\partial H}{\partial \rho} = \frac{1}{r}\left\{-v_r\frac{\partial \Lambda}{\partial \rho} - (v_\theta + w)\frac{\partial \Delta}{\partial \rho}\right\} + \frac{\partial N}{\partial \rho}\\
c_7 &= \frac{\partial H}{\partial T} = \frac{1}{r}\left\{-v_r\frac{\partial \Lambda}{\partial T} - (v_\theta + w)\frac{\partial \Delta}{\partial T}\right\} + \frac{\partial N}{\partial T}\\
c_8 &= \frac{\partial H}{\partial w} = \frac{1}{r}\left\{-v_r\frac{\partial \Lambda}{\partial w} - \Delta - (v_\theta + w)\frac{\partial \Delta}{\partial w}\right\} + \frac{\partial N}{\partial w}\\
c_9 &= \frac{\partial H}{\partial W} = \frac{1}{r}\left\{\frac{\partial \Psi}{\partial W}\cos\beta - v_r\frac{\partial \Lambda}{\partial W} - (v_\theta + w)\frac{\partial \Delta}{\partial W}\right.\\
c_{10} &= \frac{\partial H}{\partial I} = \frac{\partial N}{\partial I}
\end{aligned}
\right\} \ldots[52]
$$

After the power cut-off, the thrust f vanishes. Thus for $t > \bar{t}_1$, Ψ and its derivatives are zero.

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A Method for Comparing the Performance of Power Plants for Vertical Flight

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A new method of power plant selection for vertical flight is proposed. It can be used to determine whether the performance of a rocket design can be improved by substituting for the rocket motor a different power plant such as a ramjet. Calculations indicate that there are advantages in using the ramjet provided the power plant can be made to operate under rapid acceleration and at high altitudes.

A PROBLEM constantly facing engineers who use jet propulsion power plants is to determine the best power plant among a multitude of possible power plants for a particular design application. A very general method of power plant selection was proposed, perhaps for the first time, by W. Bollay and E. Redding, based upon the concept of lowest total installation weight. The total installation weight for a specified thrust at given altitude and speed of flight is the sum of dry power plant weight plus the weight of fuel and fuel tank for a given duration of flight at that altitude and speed. The fuel and fuel tank weight increase with the increase in flight duration. Thus, a light power plant with large fuel consumption, such as a rocket, is competitive with a heavier power plant with smaller fuel consumption only at short flight durations. This concept was extensively developed by Th. von Kármán in his general analysis of jet propulsion power plants (1).[2] This method of power plant selection was also described by A. L. Lowell (2).

However, no actual vehicle will fly at constant speed and constant altitude. There is always a definite flight plan describing the speed and altitude as functions of time. Hence, true power plant selection must depend upon a sort of weighted average of different speeds, and altitudes according to the particular flight plan of the vehicle under consideration. If the speed and altitude of the vehicle are rapidly varying, as in the case of accelerated vertical flight, then the selection based upon total installation weight at a fixed speed and a fixed altitude would be quite wrong. The purpose of the present note is to give a different method of power plant selection for vehicles in vertical flight. A meteorological sounding vehicle is a direct example of such an application. For other types of vehicle, the vertical powered flight is often a good approximation to the true flight trajectory during the application of propulsive power. Therefore the proposed method is believed to have a wider range of usefulness than is, perhaps, first apparent.

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[2] Numbers in parentheses refer to the References on page 212.

General Relation

It will be assumed in this analysis that the gravitational acceleration and the effective exhaust velocity of the power plant are constants. For moderate altitudes, the decrease of the gravitational acceleration from its sea-level value is very small and thus negligible. The effective exhaust velocity of the power plant, defined as ratio of thrust generated by the power plant to the mass rate of consumption of fuel or propellant carried within the vehicle, is of course a variable, not a constant, even for a given power plant using a given fuel or propellant. For a rocket, this variation is caused by the change in atmospheric pressure with altitude, and is usually small enough to be neglected. For a ramjet, the effective exhaust velocity increases very rapidly with speed of the vehicle in the subsonic range, but in the supersonic range the exhaust velocity is again almost constant with respect to the speed of the vehicle, within the useful speed range of the engine (3). For other power plants, the variation of the effective exhaust velocity may be more complicated; but as a first approximation, it is generally possible to use an average value as the assumed constant exhaust velocity.

For a rocket vehicle, the air drag is proportional to the cross-sectional area of the body, but the mass of the vehicle is proportional to the volume of the body. Therefore for vehicles of similar design, the air drag is proportional to the square of the body diameter, while the mass of the vehicle is proportional to the cube of the body diameter. For large rocket vehicles the drag force is then negligible with respect to the gravitational force. Calculations seem to show that for a high performance rocket of 50 tons gross weight, the air drag reduces the velocity at the end of powered flight by only 5 per cent. Hence for a large rocket vehicle, the effects of air drag on the performance can be neglected in the approximate analysis attempted here. When another power plant is used, the design cannot perhaps be made as compact as the rocket. For instance, the ramjet requires a rather large duct to produce a sizable thrust. Then the air drag may not be negligible even for a large vehicle. However, in such cases the air drag of the power plant installation can be charged against the thrust produced. As an

example, the air drag of the outside surfaces of the ramjet duct can be deducted from the thrust, produced by the ramjet, and the power plant is considered to produce a smaller "net thrust" while the body drag is considered to remain at the same magnitude without the ramjet duct. Therefore if one takes the effective, exhaust velocity as that based upon the net thrust, then the same argument for a large rocket applies and the drag of the body can be neglected for large vehicles.

For vehicles that are not so large, neglect of the air drag will certainly introduce an error. But the emphasis here is the comparison of the performance obtainable from different power plants rather than the absolute value of performance. Therefore the error made in this way is believed to be not large, and the method given below is useful even for moderate-sized vehicles.

Let m be the mass of the vehicle at time instant t when the vertical position and vertical velocity of the vehicle are $y(t)$ and $\dot{y}(t)$, respectively. Denote by c and g the constant exhaust velocity and the gravitational acceleration. Then the balance of inertia force and gravitational against the thrust (Fig. 1) gives

$$c\frac{dm}{dt} + m\left(\frac{d\dot{y}}{dt} + g\right) = 0 \quad \ldots\ldots [1]$$

If the initial velocity at $t = 0$ is zero, as is usually the case, Equation [1] integrates to

$$\log\frac{m}{m_0} = -\frac{1}{c}(\dot{y} + gt) \quad \ldots\ldots [2]$$

If the subscript 1 denotes conditions at the end of the powered flight, then Equation [2] gives

$$\log\frac{m_0}{m_1} = \frac{1}{c}(\dot{y}_1 + gt_1) \quad \ldots\ldots [3]$$

This is the fundamental performance equation for vertical flight. It has been derived previously by many authors (4), but the present deviation clearly shows that it is quite general and independent of the particular way the thrust is programmed during the powered flight.

For a vehicle intended for obtaining long range but having a vertical powered trajectory, the performance is essentially determined by the velocity at the end of powered flight or $\dot{y}_1$. For an atmospheric sounding vehicle, the summit altitude is determined by y_1 and $\dot{y}_1$. The two vehicles having the similar thrust programming during powered flight and the same $\dot{y}_1$ and t_1 will have the same y_1 and $\dot{y}_1$. To simplify the calculation, the performances of two vehicles with different power plants are made to be the same by specifying that the values of y_1, $\dot{y}_1$, and t_1 at end of the powered flight are the same for the two vehicles. This condition will be satisfied if the acceleration programs of the two vehicles are the same. The thrust programs of the two vehicles are different, however, due to the different fuel consumption of the two power plants. Therefore, one of the criteria for equal performance is the value of $\dot{y}_1 + gt_1$, occurring on the right of Equation [3]. According to Equation [3], then, to have the same performance, $c \log (m_0/m_1)$ must be the same for vehicles. This conditions will be used to compare the performance of different power plants.

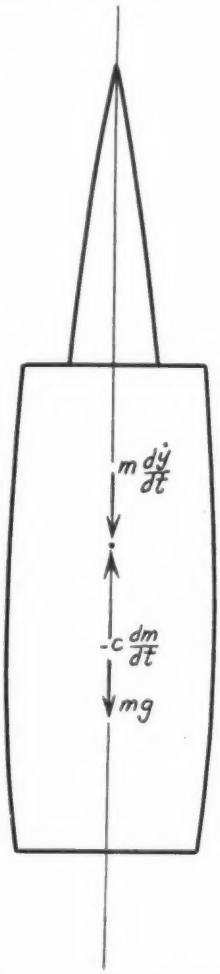

FIG. 1. BALANCE OF FORCES ON A VEHICLE IN VERTICAL ASCENT

Now let w_0, w_s, w_e, and w_1 denote the gross weight, the weight of structures and pay load, the engine weight, and the weight of the vehicle, respectively, at the end of powered flight. Then

$$w_1 = w_s + w_e \quad \ldots\ldots [4]$$

Equation [3] can be written as

$$c\log\frac{w_0}{w_s + w_e} = \dot{y}_1 + gt_1 \quad \ldots\ldots [5]$$

Consider now two vehicles, both having the same gross weight w_0; one is a rocket with $c = c^*$, $w_s = w_s^*$, $w_e = w_e^*$, and

$$\frac{w_0^*}{w_s^* + w_e^*} = \frac{1}{1 - \zeta} \quad \ldots\ldots [6]$$

where ζ is the so-called propellant loading ratio or the fraction of propellant weight in the gross weight. The other vehicle with $w_0 = w_0^*$, $w_s = w_s^*$, but with c and w_e different from c^* and w_e^*. This means that the structural weight and pay load of the vehicle with the alternate power plant remain the same as the rocket, but the engine weight is different. To compare the performance of the power plants, the maximum allowable engine weight w_e for equal performance should be calculated. If the actual engine weight is less than this calculated maximum, then the pay load can be increased over that possible for the rocket. In other words, the rocket motor can be substituted by the new power plant with a net gain in performance. If the actual engine weight is higher than the calculated maximum, then the alternate power plant will give a poorer performance than the rocket engine.

The conditions to determine the maximum allowable engine weight w_e are

$$c^*\log\frac{1}{1-\zeta} = c^*\log\frac{w_0^*}{w_s^* + w_e^*} = \dot{y}_1 + gt_1$$

$$c\log\frac{w_0}{w_s + w_e} = c\log\frac{w_0^*}{w_s^* + w_e} = \dot{y}_1 + gt_1 \quad \ldots\ldots [7]$$

Therefore by eliminating $\dot{y}_1$ and t_1, one has

$$\frac{w_e}{w_e^*} - 1 = \left(\frac{w_1^*}{w_e^*}\right)\left[(1-\zeta)^{-(1-(c^*/c))} - 1\right] \quad \ldots\ldots [8]$$

If instead of the effective exhaust velocity, the engine consumption is specified by the specific consumption s

in lb of fuel per lb of thrust per unit time, then, since s is inversely proportional to c, Equation [8] can be written as

$$\frac{w_e}{w_e^*} - 1 = \left(\frac{w_1^*}{w_e^*}\right)[(1-\zeta)^{-(1-(s/s^*))} - 1] \quad \ldots\ldots [9]$$

where s is the specific consumption of the engine under investigation and s^* the specific consumption of the comparison rocket engine. The left sides of Equations [8] and [9] are the maximum allowed increase in engine weight over the rocket engine weight divided by the rocket engine weight. The first factors on the right sides of Equations [8] and [9] are the ratio for the rocket vehicle of the weight at end of powered flight or burnout and the engine weight. The second factor on the right sides of Equations [8] and [9] are plotted in Fig. 2.

Fig. 2 shows that as the propellant loading ratio ζ for the rocket vehicle increases, the allowable increase in engine weight for a fixed reduction in specific consumption s is extended. Thus there is wider latitude in the choice of power plant for high performance vehicles than there is for a low-performance vehicle. This is certainly an encouraging fact for design engineers. It also points out the fact that the choice of power plant cannot be made independent of the performance of the vehicle but rather is intimately related to the performance of the vehicle.

Applications

As a first example, consider the possibility of using a ramjet as the propulsive power plant for a vehicle of the performance of a V-2 rocket. The weight breakdown of a V-2 is given in Table 1 (5):

TABLE 1 V-2 ROCKET

Pay load:			
Explosive charge	980 kg		
Auxiliary devices	300 kg		
		1280 kg	
Structural Engine		1750 kg	
Pumping unit	450 kg		
Rocket motor	550 kg		
		1000 kg	
Weight at burn-out			4,030 kg
Propellant and Fuel:			
Propellant		8750 kg	
Fuel for turbine		200 kg	
			8,950 kg
Gross weight			12,980 kg

Therefore $\zeta = 8950/12{,}980 = 0.69$. The ratio $w_1^*/w_e^* = 4030/1000 = 4.03$. The effect exhaust velocity including the fuel for turbine drive can be taken as $c^* = 7000$ ft/sec or $s^* = 16.56$ lb per lb thrust per hr.

For a ramjet, the fuel consumption based upon net thrust can be taken as $s = 4$ lb per lb thrust per hr (3). Then, by using Fig. 2, the maximum allowable increase in engine weight is given by

$$\frac{w_e}{w_e^*} = 6.77$$

This means that the substitute ramjet engine could weigh as much as 14,900 lb without impairing the performance of the V-2. The V-2 rocket has a thrust of 55,000 lb. This is also the average thrust to be produced by the ramjet. Then the maximum allowable ramjet weight is 0.271 lb per lb of thrust. The actual ramjet weight is probably less than this value. Therefore if the ramjet can be made to operate properly under conditions of rapid vertical acceleration and altitude variation up to high altitude, then there is advantage in substituting the ramjet for the rocket engine. This possibility has motivated more accurate calculations of a ramjet in vertical flights by L. H. Schindel (6) and J. V. Rowny (7). A ramjet as the power plant for the first stage of a high performance seems particularly attractive.

There are studies (8, 9) on the possibility of using the nuclear reactor as an energy source for rocket propulsion. In such a nuclear rocket the reactor may take the form of porous material through which the working fluid, say hydrogen, is passed and heated. A recent investigation by L. Green (10) indicated the feasibility of such a scheme. Now one of the difficulties of using nuclear reactor is the heavy weight of the reactor, particularly when a radiation shield is required. Counteracting the weight is the higher effective exhaust velocity possible. For instance, if hydrogen could be heated to 6000 F, then the data given by F. J. Malina and M. Summerfield (11) point to a value of c equal to 26,000 ft/sec or $s = 4.46$ lb per lb of thrust per hr. This

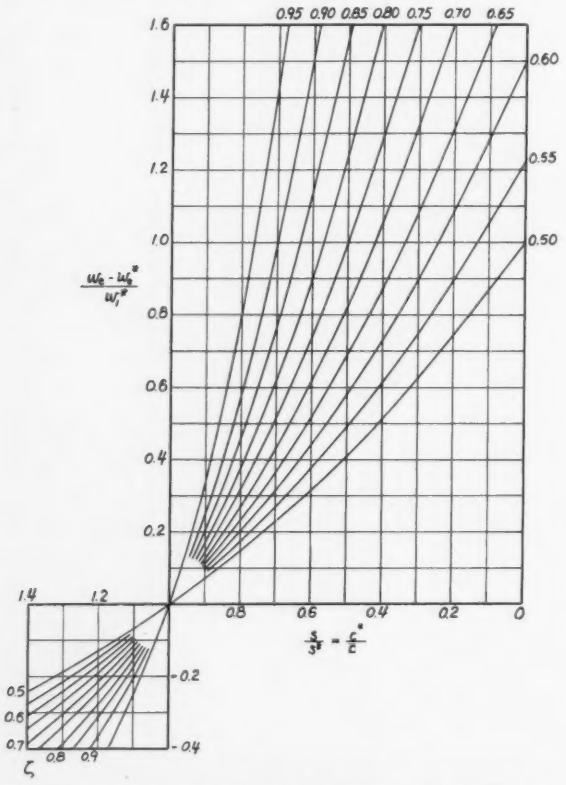

FIG. 2. ENGINE WEIGHT FACTOR AS FUNCTIONS OF ROCKET PROPELLANT LOADING RATIO ζ AND SPECIFIC FUEL CONSUMPTION s. CF. EQUATIONS [8] AND [9]

aspect of the problem of a nuclear rocket can, however, be analyzed by using the present method. For example, if the high performance rocket with energetic chemical propellant has the following specifications.

ζ = 0.80
s^* = 10 lb per lb of thrust per hr
w_1^*/w_e^* = 2.7

Then according to Equation [9], a nuclear rocket engine giving performance equal to the chemical rocket can have an engine weight 4.89 times the chemical rocket motor. In other words, if the nuclear reactor together with its necessary auxiliary construction increases the engine weight by more than 389 per cent, then the nuclear rocket is not feasible for the performance studied. If the increase in weight is not so much, then the nuclear rocket is worth while.

Detail Improvement of Rocket Engine

A rocket designer is often confronted with the problem of improving a given design at the expense of increasing the engine weight. For example, the effective exhaust velocity of a propellant can be generally increased by increasing the combustion chamber pressure. But increasing the chamber pressure would require heavier construction and an increase in the feed pressure for the propellant. Increasing the feed pressure would in turn increase the weight of the feed system. Therefore the improvement in propellant consumption is to be achieved only with higher engine weight. The present analysis can be used to determine whether such a change will or will not improve the overall performance of the vehicle.

For small changes in effective exhaust velocity or specific consumption, Equations [8] and [9] can be simplified to relate the allowable fractional increase in engine weight $\Delta w_e/w_e^*$ to fractional increase in effective exhaust velocity $\Delta c/c^*$ or specific consumption $\Delta s/s^*$. Thus, taking only the first order terms, one has from Equations [8] and [9].

$$\frac{\Delta w_e}{w_e^*} = \frac{\Delta c}{c^*}\left(\frac{w_1^*}{w_e^*}\right)\log\left(\frac{1}{1-\zeta}\right) \quad [10]$$

and

$$\frac{\Delta w_e}{w_e^*} = \frac{\Delta s}{s^*}\left(\frac{w_1^*}{w_e^*}\right)\log\left(1-\zeta\right) \quad [11]$$

These equations give the allowable fractional increase in engine weight for equal performance. If the actual increase in engine weight is less, then the modification in design improves performance, and is thus desirable. If the actual increase is more, the modification is not practical.

By taking the V-2 as an example, it can be easily calculated that 1 per cent improvement in consumption is worth 4.72 per cent increase in engine weight.

(*Continued on page 212*)

Manuscripts Invited for Presentation at the Annual Convention of the ARS

The Seventh Annual Convention of the American Rocket Society will be held at the Hotel Statler, New York, N. Y., in conjunction with the Annual Meeting of The American Society of Mechanical Engineers, November 30–December 5, 1952.

Brief abstracts should be submitted as soon as possible to:

Dr. M. J. Zucrow
Program Chairman, ARS
c/o School of Mechanical Engineering
Purdue University
Lafayette, Indiana

Please refer to page 185, this issue of the JOURNAL, for acceptable scope of subject matter. Manuscripts are to be prepared in accordance with style instructions on inside back cover.

Optimum Burning Program for Horizontal Flight[1]

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The problem of determining the optimum burning program for a jet-propelled missile in horizontal flight, which is required to attain either the maximum total range or the maximum velocity at a given range, is formulated as a problem in variational calculus. The result is found to give only a partial solution. On the basis of this result a complete solution is postulated and justified, and the whole compared with the results of a constant burning-rate program to show the advantages of the optimum burning program. Consideration is given to the questions of optimum size and configuration and optimum flight altitude.

Introduction

THE optimum performance of a jet-propelled missile can be defined in many ways. Definition is dependent upon the particular requirement such as maximum altitude, maximum horizontal distance, maximum velocity at a given point, or maximum pay load for a given range and missile size. The specific requirement must determine the nature of the investigation.

This paper is concerned with the problems of maximum horizontal range and maximum velocity at a given point. Beyond these specifications there are further restrictions. Only horizontal trajectories have been considered. The requirement of constant altitude results in constant sound speed, constant air density, and constant effective jet velocity. Further simplifying assumptions have been made; namely, the coefficient of drag at zero lift has been taken as constant, and the vertical component of the thrust has been neglected. It will be shown that only a small error is introduced by these assumptions. A similar investigation, in which the induced drag was neglected, has been carried out by Puckett and Edwards (1).[3]

Part of the importance of this form of the problem rests upon consideration of air-launched missiles, either the second stage of a two-stage missile or an aircraft-launched missile. The first type, the second stage of a two-stage missile, is usually launched at high altitude. From this launching point and for a large portion of the remaining trajectory, variations introduced by changes in altitude will be small. Therefore computations based on the assumption of constant altitude need not be very incorrect. The second type, the aircraft-launched missile, will usually be of the short-range type, and again errors stemming from the assumption of constant altitude will not be great.

Apart from these two considerations, the problem in this restricted form is capable of precise solution, and the results thereof can be interpreted without ambiguity. These results give a useful picture of the effects of various factors acting upon missiles flying on a generally horizontal trajectory, the relative importance of these effects, and the consideration which must be given to them in a more general treatment.

In addition to the usefulness of this work for the practical problems, this calculation is significant as a preliminary application of the methods of variational calculus to more general trajectory problems. The more general problems lead to such complex results that a complete investigation of the simpler cases is required in order to clarify the mathematical picture.

Equation of Motion

The general form of the equation of motion can be written in accordance with the assumptions as

$$m \frac{dv}{dt} = T - D$$

for a missile in horizontal flight, where m is the instantaneous mass of the missile, dv/dt the rate of change of the velocity with time T the thrust, and D the drag.

The drag term must have two parts, the parasite drag and the induced drag. The parasite drag can be expressed as

$$D_{\text{parasite}} = {}^1/_2 \rho C_D S v^2$$

where C_D is the coefficient of drag and will be held constant, ρ is the density of the surrounding air and is constant as noted, and S is some characteristic area of the missile, such as frontal area. The induced drag becomes

$$D_{\text{induced}} = \frac{L^2}{{}^1/_2 \pi \rho b^2 v^2}$$

where L is the total aerodynamic lift, ρ the density of the air, b a length (assumed to be constant) which for subsonic speeds is equal to the span of the lifting surface, and v the velocity (2). The requirement of horizontal flight leads to the relation $L = mg$, where m is the instantaneous mass and g the acceleration of gravity, so long as the vertical component of the thrust is neglected and the force of gravity is counteracted solely by the lift. The accuracy of this approximation will

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[3] Numbers in parentheses refer to the References on page 212.

increase with the velocity, i.e., with the decrease of attack angle. Using this value for the lift, the induced drag becomes

$$D_i = \frac{m^2 g^2}{{}^1/_2 \rho \pi b^2 v^2}$$

The thrust term can be written as

$$T = - V_e \frac{dm}{dt}$$

where V_e is the effective jet velocity (which is constant for constant altitude) and dm/dt is the rate of consumption of fuel with time.

Now the complete equation becomes (dividing through by mv)

$$\frac{dv}{dx} = - V_e \frac{1}{m} \frac{dm}{dx} - {}^1/_2 \rho S C_D \frac{v}{m} - \frac{2g^2 m}{\pi \rho b^2 v^3}$$

where x is the distance traveled by the missile.

The variables appearing in this equation can be made dimensionless in several ways. Choosing a special one of these, namely,

$$\bar{m} = \frac{2mg}{V_e^2 \rho b \sqrt{\pi C_D S}}$$

$$\bar{v} = \frac{v}{V_e}$$

$$\bar{x} = x \frac{g}{b V_e^2} \sqrt{\frac{C_D S}{\pi}}$$

$$\bar{t} = t \frac{g}{b V_e} \sqrt{\frac{C_D S}{\pi}}$$

the equation of motion can be written as

$$\frac{d\bar{v}}{d\bar{x}} = - \frac{1}{\bar{m}} \frac{d\bar{m}}{d\bar{x}} - \frac{\bar{v}}{\bar{m}} - \frac{\bar{m}}{\bar{v}^3}$$

The following conventions will be used to indicate special points on the trajectory:

( )$_0$ = initial conditions.
( )$_b$ = conditions at burnout.
( )$_f$ = conditions at the termination of the horizontal flight.

This last point will occur when the induced drag term becomes so large, because of the decrease in velocity, that the missile is prevented from maintaining a horizontal trajectory and stalls. Thus $v_f = \bar{v}_f = 0$, and $\bar{m}_b = \bar{m}_f =$ the dimensionless empty mass of the missile.

Optimum Burning Program

A Application of the Calculus of Variations

The distance traveled by the missile while burning, an amount of fuel equal to $\bar{m}_1 - \bar{m}_2$, can be expressed as the integral (in dimensionless form)

$$\bar{x} = - \int_{\bar{m}_1}^{\bar{m}_2} \frac{(d\bar{v}/d\bar{m}) + (1/\bar{m})}{(\bar{v}/\bar{m}) + (\bar{m}/\bar{v}^3)} \, d\bar{m}$$

The calculus of variations may be applied to this integral (3). The integrand may be written as

$$\frac{\bar{v}' + (\bar{m})^{-1}}{\bar{v}(\bar{m})^{-1} + \bar{m}\,(\bar{v})^{-3}} = \bar{v}' \, U + V = F$$

where

$$\bar{v}' = \frac{d\bar{v}}{d\bar{m}}$$

Both U and V are independent of v'.

The Euler condition which the integrand must satisfy if the integral is to have its maximum or minimum value is

$$\frac{\partial F}{\partial \bar{v}} - \frac{d}{d\bar{m}} \left(\frac{\partial F}{\partial \bar{v}'} \right) = 0$$

With F in the form just given, the condition becomes

$$\frac{\partial V}{\partial \bar{v}} - \frac{\partial U}{\partial \bar{m}} = 0$$

Substituting for U and V, this expression is

$$\frac{\bar{v}}{\bar{m}^2} - \frac{1}{\bar{v}^3} + \frac{1}{\bar{m}^2} - \frac{3}{\bar{v}^4} = 0$$

or

$$\bar{m} = \bar{v}^2 \left(\frac{\bar{v} + 1}{\bar{v} + 3} \right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Equation [1] is the relation which the variables of mass and velocity must satisfy if the integral is to have either a maximum or a minimum value. The result is limited in application. Its algebraic nature prevents arbitrary assignment of initial values to both the mass and the velocity. Since the variables must satisfy this relation at all points, only one can be chosen at will for the initial point. Therefore questions arising from independent and arbitary choice of initial conditions are not answered by this relation. Furthermore, there is no chance to apply other tests of variational calculus, such as the Legendre test, to determine whether or not this integral can have a maximum or a minimum. Separate consideration must be given to initial points, and some method must be devised to determine whether or not a maximum value of the range is obtained by following this relation.

If this relationship is rewritten as

$$(\bar{m})^2/(\bar{v})^2 = (\bar{v}) \left(\frac{\bar{v} + 1}{\bar{v} + 3} \right)$$

it can be interpreted as a relationship between the induced drag $\bar{m}^2/\bar{v}^2$ and the parasite drag $\bar{v}^2$. A similar relationship holds for the airplane (2), where the maximum range will be obtained if parasite drag is held equal to induced drag. In the present considerations for rocket-propelled missiles, the factor of proportionality between the two drag terms varies between unity for infinite velocity and $^1/_3$ for zero velocity.

B Problems at the Initial Point

The problems resulting from the arbitrary choice of initial values for the mass and velocity can be divided into three types. Suppose that the initial mass has been predetermined; then its value, together with Equation [1], will determine the optimum initial velocity. But suppose the initial velocity is also predetermined; then one of the three types of problems will arise depending on whether the predetermined initial velocity is higher than, equal to, or lower than the optimum.

Only the first and third types present any real problem since in the second, when the initial velocity is equal to that required by the optimum relationship, the optimum burning program can be begun immediately. The first type has only small practical significance since most missiles will begin the horizontal portion of their trajectory with a relatively small horizontal velocity. In any event, if the velocity of the missile is at any time greater than that required by Equation [1], at some previous time the velocity of the missile or its launching vehicle must have been equal to the optimum value and, at an earlier time, lower than optimum.

A burning program is postulated, namely: Sufficient fuel is burned in an instantaneous pulse to raise the velocity and lower the mass to those values which satisfy Equation [1]. Thereafter the optimum conditions of Equation [1] are maintained. It is assumed that this program will lead to the maximum range of the missile. This assumption is justified by a direct comparison between the values of range obtained in this manner and those which could be obtained in any other manner.

The change in velocity resulting from an infinite burning rate (i.e., $d\bar{m}/d\bar{t} = -\infty$) can be derived from the equation of motion by setting the change in distance equal to zero. The velocity $\bar{v}_1$, after an amount of fuel $\bar{m}_0 - \bar{m}_1$ has been burned, is then

$$\bar{v}_1 = \bar{v}_0 + \log_e \frac{\bar{m}_0}{\bar{m}_1}$$

By requiring m_1 and v_1 to satisfy Equation [1], the values of mass and velocity in terms of their initial values may be determined. At this point the missile has traveled a zero distance and is prepared to begin flight according to the optimum relation of Equation [1]. Now consider any small deviation from this burning program, which may be characterized by the requirement that a small amount of fuel $\Delta\bar{m}$ must be burned at a noninfinite rate. Thus $d\bar{m}/d\bar{t} = -M > -\infty$ when the mass $\bar{m}_2$ and velocity $\bar{v}_2$ of the missile are such that

$$\left.\begin{array}{l} \bar{m}_0 \geq \bar{m}_2 \geq \bar{m}_1 \\ \bar{v}_0 \leq \bar{v}_2 \leq \bar{v}_1 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [2]$$

During the burning of this small portion of the fuel the missile will travel a distance $\Delta\bar{x}'$ which can be computed in terms of the initial values of mass and velocity $\bar{m}_0$ and $\bar{v}_0$ and the value of the mass $\bar{m}_2$ at that point where the small deviation from the infinite burning rate took place. After this small deviation, the infinite burning rate is re-established and continued until optimum conditions are reached. The values of mass and velocity at this point are designated as $\bar{m}_1'$ and $\bar{v}_1'$. It can be shown (4) that $\bar{m}_1 > \bar{m}_1'$ and $\bar{v}_1 > \bar{v}_1'$. Therefore, to complete the comparison, the flight of the missile which followed the proposed optimum burning program without deviation is continued on the optimum course until its mass and velocity are equal to the values $\bar{m}_1'$ and $\bar{v}_1'$ obtained by following the deviated course. The distance traveled $\Delta\bar{x}$ for this portion of the flight can be computed. The ratio between the two ranges obtained by following these two burning programs can then be determined as a function of m_0, v_0, and k, where k is given by

$$k = \frac{\bar{m}_2}{\bar{m}_1}$$

and by Equation [2], must be greater than or equal to unity. Thus

$$\frac{\Delta\bar{x}}{\Delta\bar{x}'} = f(\bar{m}_0, \bar{v}_0, k)$$

When k is equal to 1, this ratio is equal to 1. For all values of k greater than 1, it can be shown that the derivative of this ratio with respect to k is positive. Therefore the ratio is a never-decreasing function of k and has the value 1 when k equals 1.

Since any finite burning program can be approximated as closely as desired by a finite number of such small deviations from an infinite burning program, the assumption is proved.

Thus, if the initial mass and velocity are such that

$$\bar{m}_0 > \bar{v}^2{}_0 \left(\frac{\bar{v}_0 + 1}{\bar{v}_0 + 3}\right)^{1/2}$$

burning fuel at an infinite rate until optimum conditions are established will lead to a greater range than will any other burning program for the same fuel consumption. Of course an infinite burning rate is not practically possible, but an initial impulse similar to that of a gunshot or a high-acceleration boost would be a feasible approximation of the requirements.

The problem of an initial velocity which is higher than that required by the assigned initial mass and Equation [1] can be treated in a similar manner. It can be shown that the maximum range will result from following a coasting flight until the velocity has dropped to the optimum value and then continuing flight according to Equation [1]. The details are not included in this paper. These results lead to a general conclusion: Burning fuel while moving on any trajectory other than that determined by Equation [1] will prevent the missile from attaining its maximum possible range in horizontal flight. If the initial velocity is too high, the missile must be allowed to move without any fuel consumption until optimum conditions are established. If the initial velocity is too low, fuel must be burned with an infinite burning rate until optimum conditions are established.

C Four Problems Which Arise During Powered Flight

The powered-flight portion of the trajectory, in general, has already been considered under the variational calculus. But some problems arising near the initial and final points of the powered flight cannot be adequately handled by variational means and must be given special consideration. There are four questions to be answered:

1 When the initial velocity is lower than optimum, it has been shown that the initial burning rate should be infinite. This type of burning must be continued at least until optimum conditions are established. Perhaps it should be continued beyond this point. No

previous calculations have indicated the contrary. The question then is: As soon as optimum conditions have been established by an infinite burning rate, what burning program should be applied next?

2 When the initial velocity is higher than optimum, it can be shown that a coasting flight must be maintained until optimum conditions are established. Then the question is: As soon as optimum conditions have been established by a coasting flight, what burning program should be applied next?

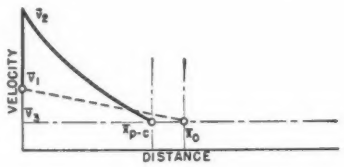
3 The variational treatment has considered only the powered flight. The total range will comprise both the powered flight and the subsequent horizontal coasting flight. No previous calculations have indicated that the burning program which results in the maximum range during the powered flight will produce the maximum total range. Since the distance traveled while coasting will be greater for greater burnout velocities, some alternate burning program which produces greater burnout velocities than the optimum program might result in the greatest total range. Any alternate program will, of course, result in a smaller range for the powered flight. Therefore the increase in coasting range produced by such an alternate program must be greater than this resultant decrease in powered-flight range. The question then is: Is there any burning program which fulfills these requirements?

4 The maximum possible velocity at a given range may be required. If the range given in this requirement is greater than the maximum possible powered-flight range, the requirement will be met by that burning program which results in the greatest total range. However, if the given range can be reached before the end of the powered flight, the optimum burning program so far outlined will not give the greatest possible velocity. This fact is immediate since the missile will attain its prescribed range while some unburned fuel remains. This fuel could be used to increase the velocity. The question then is: What is the burning program which will result in the maximum possible velocity at a given range, assuming that this given range can be reached on the optimum trajectory (defined by Equation [1]) before the fuel is exhausted?

In answer to Question 1, it can be shown that, as soon as optimum conditions have been established, the infinite burning rate must stop and be succeeded by a burning rate which will cause the mass and velocity to maintain the relationship defined in Equation [1], the optimum condition. In demonstrating this fact, only one alternative to the optimum rate need be considered, i.e., continuation of the infinite burning rate. Any noninfinite burning rate which starts with values of mass and velocity satisfying Equation [1] is excluded by the results of the variational treatment. Only the infinite rate fails to answer the continuity requirements necessary for comparison by means of the calculus of variations. Therefore assume that the infinite burning rate has been continued past that point at which the mass $\bar{m}_1$ and velocity $\bar{v}_1$ satisfy Equation [1]. The resulting velocity is higher than that required by the optimum relation. Suppose that the burning is now stopped at some arbitrary value of mass $\bar{m}_2$. The velocity $\bar{v}_2$ at this point can be expressed as a function of $\bar{m}_1$, $\bar{v}_1$, and $\bar{m}_2$

$$\bar{v}_2 = \bar{v}_1 + \log_e \frac{\bar{m}_1}{\bar{m}_2}$$

It has been pointed out that in such a case the best burning program to follow consists of a coasting flight until the velocity has dropped to its optimum value. The answer to Question 1 will then be found in a comparison between (*a*) the range obtained from instantaneous-pulse burning followed by a coasting flight (initiating and terminating at optimum conditions), and (*b*) the range obtained by following these optimum conditions between the same initial and terminal points of mass and velocity. Sketch A, a diagram of velocity



SKETCH A

plotted against distance, gives a qualitative picture of this situation.

The difference between the ranges is made a function of $\bar{v}_1$ (the initial velocity of the two ranges) and a number k which is defined as the ratio of the initial to final velocities. The expression of this difference is

$$D = \bar{x}_0 - \bar{x}_{p-c} = D(v_1, k)$$

where $\bar{x}_0$ is the range obtained from following the optimum burning program, and $\bar{x}_{p-c}$ is that obtained by burning the same amount of fuel in an instantaneous pulse and then coasting to the same terminal velocity. It can be shown that this difference is zero when k is unity and that the differential of this difference with respect to k is positive for all values of k equal to or greater than 1. Thus the range obtained by consuming any amount of fuel according to the optimum burning program will exceed the range obtained by burning this same amount of fuel in an instantaneous pulse and then coasting when both programs have the same initial and final values of mass and velocity.

In Question 2 the conditions state that the optimum relation between mass and velocity determined by Equation [1] is established by coasting in horizontal flight from a velocity higher than optimum. If burning is to commence at this point, the rate of such burning must be the optimum rate. The infinite rate is excluded by the answer to Question 1. Any noninfinite rate which is other than optimum is excluded by the calculus of variations. Therefore there are only two specific alternatives. One is the continuation of the coasting flight; the other is the commencement of burning according to the optimum requirements.

As in the treatment of Question 1, expressions for the distance traveled by the missile along each of the two alternate flight programs are developed, and their difference is considered. Suppose that the velocity at the point where optimum conditions are first established is $\bar{v}_1$. Following the coast-pulse program from this point, the missile is allowed to coast until some velocity $\bar{v}_2'$ is reached. This velocity is arbitrary save for the requirement that it must not be so low that there is insufficient fuel to raise the velocity to optimum by instantaneous-pulse burning. Since the missile must maintain a horizontal trajectory, the only forces acting during this portion of the flight are drag forces. Therefore $\bar{v}_1 > \bar{v}_2'$. Since the velocity is now lower than optimum, previous considerations state that the optimum use will be made of the fuel if a sufficient amount of it is burned instantaneously to raise the velocity and lower the mass until optimum conditions are established. Suppose that this requirement is fulfilled. The velocity $\bar{v}_3$ at the end of this coast-pulse program is then optimum.

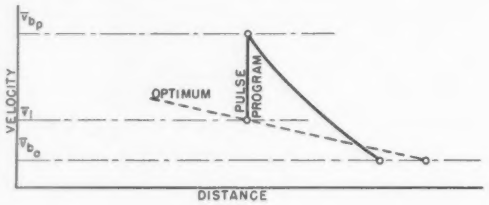
The alternative is the optimum program. Herein, as soon as the velocity $\bar{v}_1$ is reached, burning according to the optimum program is initiated and continued until the missile has reached a velocity $\bar{v}_3$. Now the two alternate programs have the same initial velocities $\bar{v}_1$ and final velocities $\bar{v}_3$. Both these velocities satisfy Equation [1]. Therefore both programs have the same initial mass $\bar{m}_1$ and final mass $\bar{m}_3$. The distances traveled in the two programs and the difference between them are made a function of $\bar{v}_1$ and a number k', defined as the ratio of the final to the initial velocity and therefore always less than or equal to 1.

The difference is zero when k' equals 1, i.e., when the missile is allowed to coast for zero time on the coast-pulse program. The parameter k' can be understood physically as a measure (a) of the length of time the missile is allowed to coast on the coast-pulse program or (b) of the strength of the instantaneous pulse necessary to return the velocity to optimum at the end of coasting flight. Thus the time (or the strength of the final pulse) is zero when k' equals 1, and the initial and final velocities are identical. The time of coasting then increases as k' decreases.

The differential of the difference between the distances traveled with respect to k' can be shown to be always equal to or less than zero for k' equal to or less than unity. Therefore the difference, which is zero when k' is 1, can only increase as k' decreases. Thus such a coast-pulse program which starts and ends at optimum conditions can never result in a greater range than the optimum program.

Question 3 arises at the end of powered flight. It is necessary to determine whether the optimum burning program will result in the greatest total range as well as the greatest burning range. Therefore it is necessary to consider other burning programs which might sufficiently raise the burnout velocity, thus increasing the coasting range, in order to offset any loss in burning range entailed by such a deviation from the optimum burning program.

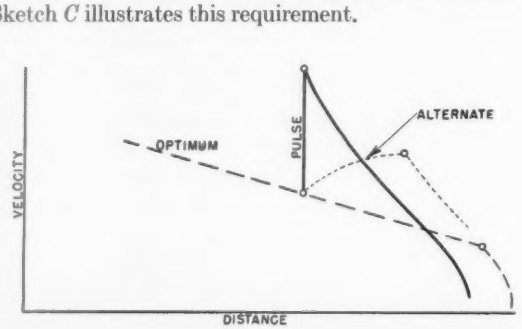
Consider an instantaneous-pulse program; i.e., after the optimum burning program is followed to some point before burnout, the remainder of the fuel is burned as an instantaneous pulse. After fuel is exhausted, the missile coasts. The burnout mass in both cases is the same, whereas the burnout velocity $\bar{v}_{b_p}$ after the pulse burning program is greater than that for the optimum program $\bar{v}_{b_o}$. Thus, after burnout of the pulse program, the velocity will decrease in the coasting portion of the flight until, in particular, it is the same as the burnout velocity for the optimum burning program. Sketch B illustrates the situation.



SKETCH B

At this velocity on each trajectory the mass and velocity for one trajectory are equal to the mass and velocity for the other. The ranges for the two trajectories will not necessarily be equal. From these two values of range (one on each trajectory), continuing to the end of the flight, both trajectories must cover the same distance. Thus any difference in total range must arise in those portions of the two trajectories which begin when the mass = $\bar{m}_1$ and the velocity = $\bar{v}_1$ (at the commencement of the instantaneous-pulse burning program), and which end when the mass = $\bar{m}_b$ (the burnout mass) and the velocity = $\bar{v}_{b_o}$ (the burnout velocity for the optimum case). But this comparison is exactly that considered in the answer to Question 1, where it was shown that the optimum burning program results in the greater range.

The instantaneous-pulse burning program then is not an acceptable alternate to the optimum burning program when maximum total range is demanded. Any burning program which might result in a greater total range than the optimum must at least give a greater burnout velocity than the optimum program. Therefore the burning rate of such a program must be something between the optimum and infinite. Sketch C illustrates this requirement.



SKETCH C

Again, two programs are considered. First, starting from some optimum conditions characterized by mass, velocity, and distance parameters $\bar{m}_1$, $\bar{v}_1$, and $\bar{x}_1$, respectively, a small amount of fuel $-\Delta\bar{m}_1$ is burned according to the optimum relationship, and a distance $\Delta\bar{x}_1$ is traveled in so doing. At this point the values $\bar{m}_2$, $\bar{v}_2$, and $\bar{x}_2$ characterize the missile. Then another small amount of fuel $-\Delta\bar{m}_2$ is burned at an infinite rate. The burning of the fuel raises the velocity to $\bar{v}_3$ without changing the range value $\bar{x}_2$. Second, an alternate program is considered. A particular non-infinite burning rate $-M$ is chosen. Thus $d\bar{m}/d\bar{t} = -M$ such that, when an amount of fuel equal to $-\Delta\bar{m}_1 - \Delta\bar{m}_2$ is burned at this rate, the resulting velocity will be equal to $\bar{v}_3$. The range traveled in this process is then calculated. This range is less than the range resulting from the first of the two programs. Any non-infinite burning program can be approximated as closely as desired over a sufficiently small interval by such a constant rate. Thus a combination of optimum and infinite-rate burning which terminates in a particular velocity and consumes a particular amount of fuel will result in a greater range than will any other burning rate which terminates in the same velocity and consumes the same amount of fuel. Similarly this combination of optimum and infinite-rate burning will result in a higher velocity at a given range than will any other rate consuming the same amount of fuel. The result has been demonstrated for only small intervals, but any larger interval can be constructed as a series of such small intervals; the results will still hold. It has already been shown that, if fuel is burned at an infinite rate at any time after optimum conditions have been established and before burnout occurs, the resulting total range will be less than the possible maximum.

Question 4 involving the method of attaining the maximum possible velocity at any assigned range has also been answered by the considerations just outlined. The burning program to be followed in order to answer such a requirement is twofold: First, the mass and velocity must be held to satisfy Equation [1] until the assigned value of range is reached. Second, the remaining fuel must be burned at an infinite rate (or as close to infinite as is practically possible).

The general conclusion noted earlier may be amplified. If maximum range is desired, the optimum relation between mass and velocity as given by Equation [1], once established, must be maintained until all fuel is exhausted. If maximum velocity at a preassigned range is desired, this same optimum relation, once established, must be maintained until the assigned range is reached. The remaining fuel is then to be burned as a pulse of zero duration.

Comparisons Between Optimum Burning Program and Other Possible Programs

It has been pointed out that the results of the variational treatment are such that no further variational tests may be applied to determine whether or not Equation [1] results in a true maximum. It is reasonable to state that, since the integrand of the range integral has been made to satisfy the Euler condition, the value of the integral is at least a maximum or a minimum or some stationary point. One method of determining which of these possibilities is correct is to compare the range obtained by following the requirements of Equation [1] with ranges obtained from different burning programs. A variety of constant burning rates will serve this purpose. If the range obtained from the optimum trajectory is greater than that obtained from any constant burning rate, the second possibility (that the calculus has produced a minimum) is discarded. Furthermore, favorable results of this comparison, together with the other computations presented in this paper would strongly indicate that a true maximum has been found.

The results of this comparison are presented in Figs. 1 through 6. Figs. 1 and 2 are graphs of velocity vs. distance for various constant burning rates and the optimum burning program. The two graphs differ in the value of initial velocity. For both values the initial velocity and mass satisfy Equation [1] for all the burning programs. Two mass-fuel ratios have been considered for each value, 60 and 70 per cent. In all cases, the optimum burning program produces the greatest range.

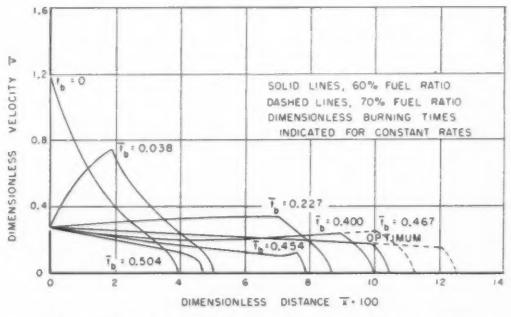


FIG. 1 RANGES OBTAINED FROM VARIOUS CONSTANT BURNING RATES COMPARED WITH RANGE OF OPTIMUM PROGRAM FOR LOW INITIAL VELOCITY

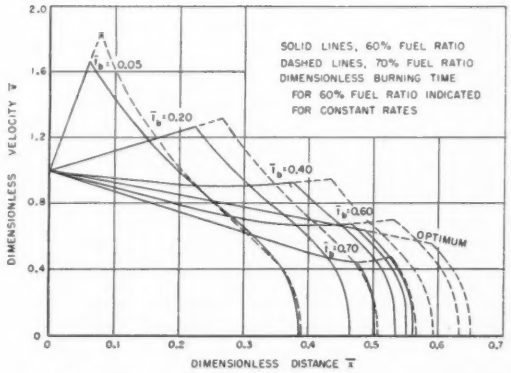


FIG. 2 RANGES OBTAINED FROM VARIOUS CONSTANT BURNING RATES COMPARED WITH RANGE OF OPTIMUM PROGRAM FOR HIGH INITIAL VELOCITY

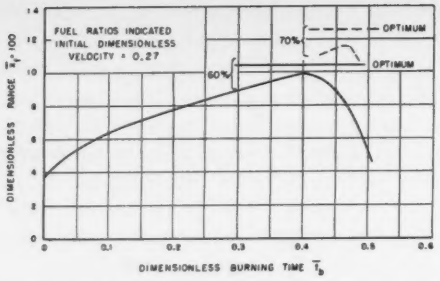


FIG. 3 TOTAL RANGE VS BURNING TIME FOR CONSTANT BURNING RATES FOR LOW INITIAL VELOCITY

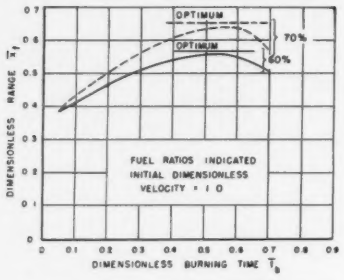


FIG. 4 TOTAL RANGE VS BURNING TIME FOR CONSTANT BURNING RATES FOR HIGH INITIAL VELOCITY

The slowest burning rate considered results in a trajectory whose velocity decreases to zero before all the fuel is used. If the vertical component of thrust has been taken into account, this stalling phenomenon would not be observed for such a burning rate but only for a rate so low that the thrust was less than the force of gravity acting on the missile. However, at higher speeds (resulting from higher burning rates) this approximation does not introduce such an error. The induced drag does not become too large.

The velocity for the optimum burning program does not vary widely; thus the assumption of constant drag coefficient has not introduced the error that might have been expected.

Figs. 3 and 4 are graphs of total range against burning time for the various constant burning rates. It is seen that one of these burning times (and thus one burning rate) results in the maximum total range. This time is different for the two different initial conditions, but the maximum thus attained is in all cases less than the total range obtained from the optimum program. This range value is indicated by a straight line on the graphs.

All variables are presented in dimensionless form; therefore several interpretations can be given to the two values of initial velocity assigned, one for Figs. 1 and 3 and the other for Figs. 2 and 4. It was stated that the initial mass and velocity satisfy Equation [1]. Thus, in terms of the actual values of the variables, the two initial conditions imply (*a*) two different initial velocities (and two different altitudes) for the same missile, or (*b*) two missiles, differing in mass, effective jet velocity, or aerodynamic characteristics, whereas the actual initial velocity and altitude remain the same. Any change in the dimensionless range introduced by these changes in physical parameters would of course be the same for the constant burning rates and the optimum program. The results would not be altered.

Figs. 5 and 6 present a comparison between the optimum burning program and various constant burning rates when the initial velocity is lower than optimum. This initial velocity requires, for the optimum program, that a certain amount of fuel be burned at an infinite rate to establish optimum conditions. In this case, the favorable results of the optimum program are even more pronounced. Neglecting the vertical component of thrust is actually quite important in this example because the induced-drag term is quite large at this low initial velocity. The lowest burning rate results in a stall very soon after launching when only a small portion of the fuel has been burned. This occurrence is unreasonable in view of the high burning rate considered.

The effect of varying initial conditions has been considered numerically, and the results are presented in Fig. 7. In this consideration two values of initial velocity are assigned, and the initial mass is allowed to vary. The total range resulting from the optimum burning program is computed, and the results are plotted against the values of initial mass. Again all variables are in their dimensionless form. The curves start at a point called the equilibrium point, corre-

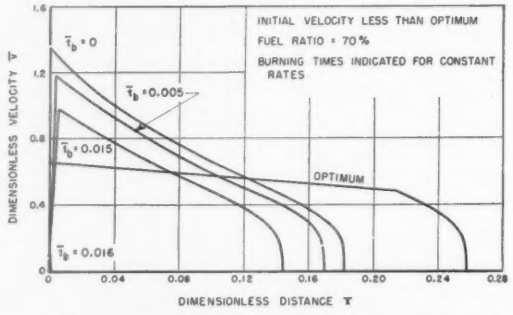


FIG. 5 RANGES OBTAINED FROM VARIOUS CONSTANT BURNING RATES COMPARED WITH RANGE OF THE OPTIMUM PROGRAM FOR A LOW INITIAL VELOCITY

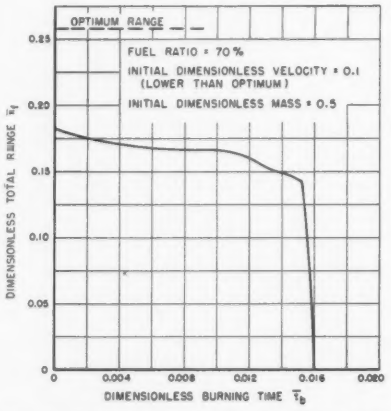


FIG. 6 TOTAL RANGE VS BURNING TIME FOR CONSTANT BURNING RATES WITH LOW INITIAL VELOCITY

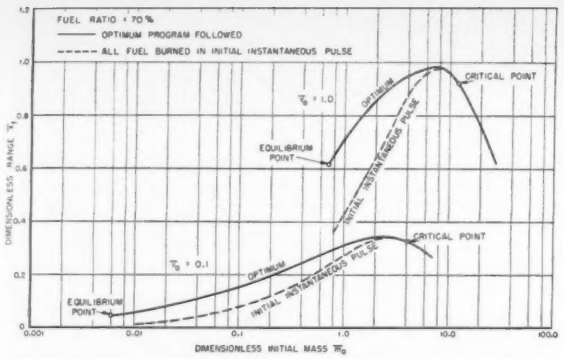


FIG. 7 EFFECT OF INITIAL DIMENSIONLESS MASS ON TOTAL RANGE

sponding to that value of mass which, together with the assigned initial velocity, satisfies Equation [1]. The curves are continued through the critical point, corresponding to that value of initial mass for which all the fuel must be burned as an infinite pulse in order to establish optimum conditions. After these conditions have been established, no more fuel remains for further powered flight. For all values of mass above this critical point, all the fuel must be burned at an infinite rate, but optimum conditions can never be established. As might be expected, each curve has a maximum range value for a particular value of initial mass. These mass values are different for each of the two curves but have a certain similarity. For both curves it was assumed that 70 per cent of the initial mass was fuel. For one curve an initial dimensionless velocity of $^1/_{10}$ was chosen, with unity for the initial velocity of the other curve. For that curve with an initial physical velocity equal to $^1/_{10}$ the effective jet velocity, it was found that the maximum range occurred for that value of initial mass such that 94.5 per cent of the fuel must be burned at an infinite rate initially. For the other curve, when the initial velocity was equal to the effective jet velocity, the maximum range was obtained from that value of initial mass such that 92.2 per cent of the fuel must be burned as an initial pulse.

From a practical point of view, it would be advantageous to find a simple, single-stage burning program which would result in a range closely approaching the maximum. The evidence just given suggests an infinite pulse burning with particular values for the initial velocity and mass. To check the results of such a burning program, the total range obtained by burning all the fuel in an instantaneous pulse was plotted as dashed lines on Fig. 7. It is seen that the range for this alternate burning program, although well below the optimum range for low values of initial mass, has a maximum differing little from the maximum computed from optimum requirements for each of the two initial velocities. The computations on which the graphs are based showed this difference to be less than 1 per cent. The values of initial mass at these maxima are approximately the same for the infinite-rate burning as for the optimum program.

Since the initial mass referred to is a dimensionless quantity, varying this quantity does not necessarily imply varying the actual mass of the missile. The altitude of the flight, the size of the missile, its aerodynamic configurations, or its effective jet velocity could be varied to produce a change in the dimensionless form of the mass.

Conclusions

A General Rules

Two burning programs have been devised to apply to a missile following a horizontal trajectory at any altitude for arbitrary initial mass and velocity. The first of these programs will result in the greatest possible total range for the given initial conditions. The second will result in the greatest possible velocity at some given range for the given initial conditions.

An introductory study of the effect of initial conditions and missile characteristics has been made, and the results are presented in graphical form (Fig. 7). These results can be outlined in three statements:

1. For a given launching velocity with a given missile, there is one altitude at which the missile will attain its greatest possible horizontal range by following the optimum burning program. It will attain a range not significantly different from this maximum if all the fuel is burned initially at an infinite rate. At any altitude above or below this optimum altitude, the total range attainable, even by following the optimum burning program, will be less than this maximum.
2. For a given launching velocity at a given altitude, there is a particular size (for a missile of given configuration and effective jet velocity) which will result in the greatest possible range when the optimum burning program is followed. The other points of Statement 1 apply here if "size" is substituted for "altitude."
3. Similar results are obtained for variations of the effective jet velocity and aerodynamic characteristics with a given initial velocity, flight altitude, and missile weight.

B Theoretical Example

A numerical example will serve to illustrate these statements. The values used are for a theoretical second-stage missile. The density at various altitudes is taken as standard (5). The velocities considered will be supersonic; therefore the number b^2, which appeared in the induced drag term and the expressions for the dimensionless variables, will have the form $b^2 = 1/\pi\ \partial C_L/\partial\alpha S$ where $\partial C_L/\partial\alpha$ is the rate of change of the coefficient of lift with attack angle based on the area S. The value $\partial C_L/\partial\alpha$ will be taken as constant. The characteristics of the missile are then as follows:

Gross initial weight	22 slugs
Weight empty	6.6 slugs
Fuel ratio	70%
Frontal area	1 sq ft
$\partial C_L/\partial\alpha$	7
C_D	0.2
Effective jet velocity	6600 ft/sec
Initial velocity	6600 ft/sec

The constant b is then equal to 1.5 feet. The air density ρ will be allowed to vary in the determination of initial conditions. The initial dimensionless velocity $\bar{v}_0$ is 1, and the initial dimensionless mass as a function of air density is

$$\bar{m}_0 = \frac{2.73 \times 10^{-5}}{\rho}$$

The final range in statute miles may be expressed as a function of the dimensionless total range

$$x_f = 1.50 \times 10^2 \bar{x}_f \text{ statute miles}$$

The maximum possible total range for any altitude is indicated by Fig. 7. The results are summarized in Table 1. The greatest of these possible values is 1470 miles and will be reached at a flight altitude of 148,000 feet.

TABLE 1 MAXIMUM POSSIBLE RANGE OF THEORETICAL SECOND-STAGE MISSILE[a]

Altitude (ft)	Range (statute miles)
100,000	965
120,000	1310
140,000	1450
148,000	1470
160,000	1380
180,000	1050
200,000	675

[a] Various launching altitudes are considered. The launching velocity is equal to the effective jet velocity. The trajectory is horizontal.

If the launching velocity were $^1/_{10}$ the effective jet velocity, the greatest attainable range for the same missile would be 417 miles at the optimum altitude of 130,000 feet.

C Recommendations

Two recommendations for further investigation result from these conclusions: First, the problem could be generalized further. It has been stated that during burning, according to the relation expressed in Equation [1], there is only a small variation in velocity; hence the assumption of a constant drag coefficient does not lead to great inaccuracy for this portion of the trajectory. However, if the initial velocity is much smaller than optimum, a large velocity change is introduced at the beginning of the flight. During this change a constant drag coefficient might be quite inaccurate. Again at the end of the trajectory, during the coasting phase, large changes in velocity occur which further limit the use of a constant coefficient of drag. Therefore in consideration of the complete trajectory it would be wise to investigate the effect of a variable coefficient, one which more closely approximated the true form. Similarly, the area b^2 which determines the induced drag and which was assumed constant is actually a slowly varying function of the flight Mach number. The more realistic nonhorizontal trajectories should certainly be considered. A solution to the problem of obtaining maximum range from trajectories which are not limited to constant altitude would find a more practical application.

Second, it has been mentioned that design characteristics of the missile enter into the determination of those initial conditions which result in the maximum possible range. A further study of the effect of these characteristics would help to determine the various criteria which must be satisfied by missiles so that they may operate efficiently at specific altitudes and with specific launching velocities.

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A Method for Comparing the Performance of Power Plants for Vertical Flight

(*Continued from page 203*)

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10 "Gas Cooling of a Porous Heat Source," by L. Green, Jr., *Journal of Applied Mechanics*, vol. 19, June 1952, pp. 173–178.

11 "The Problem of Escape From the Earth by Rocket," by F. J. Malina and M. Summerfield, *Journal of the Aeronautical Sciences*, vol. 14, 1947, pp. 471–480, particularly Table 2.

The Effect of Variable Propellant Density On Rocket Performance

By H. S. SEIFERT[1]

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Under certain rather restricted conditions, the thrust of a rocket may be increased by replacing part of the fuel by a dense inert additive. This thrust increase will not, however, increase the acceleration of a rocket vehicle of high loading factor, but may increase the acceleration of a large fixed load such as an airplane. It is difficult to see, however, how the use of such a high density fluid (mercury, for example) could be practical. This analysis is offered in reply to a previous paper published in this Journal in which advantages were claimed for the use of dense inert additives.

Nomenclature

a_0 = vertical acceleration (general)
a_1 = horizontal acceleration with pure propellant
a_m = horizontal acceleration with mixed propellant
c_1 = specific heat of pure propellant
c_2 = specific heat of inert additive
$\bar{c} = \dfrac{m_1c_1 + m_2c_2}{m_1 + m_2}$ = average specific heat of mixture
F = thrust (general)
F_1 = thrust with pure propellant
F_m = thrust with mixed propellant
g = acceleration of gravity
$K = m_2/m_1$ = mass ratio; additive to propellant
m_0 = total initial mass of propellant mixture
m_1 = total mass of pure propellant
m_2 = total mass of inert additive
N = ratio, aircraft weight to weight of ATO unit using pure propellant
T_0 = equilibrium flame temperature of pure propellant
T_m = equilibrium flame temperature of mixture
u = exhaust velocity after complete expansion
V_0 = volume of propellant tank
$\alpha = c_2/c_1$ = specific heat ratio; additive to propellant
$\beta = \rho_2/\rho_1$ = density ratio; additive to propellant

Introduction

THE suggestion was once made in an early issue of this journal[2] that the terminal velocity of a rocket might be increased if a dense fluid (for example, mercury) were mixed with the propellant during the initial part of the flight. It is the present writer's belief that this conclusion is open to question. It is interesting, therefore, to discuss anew the effect of an inert and dense additive to the propellant on the performance of a rocket.

Let us assume for simplicity that a monopropellant is being pumped into a combustion chamber at a constant *volume* rate of flow $\dot{V}_0 = dV_0/dt$. We will consider ourselves free to fill the total fixed tank volume V_0 with a mixture, in any proportion, of propellant of density ρ_1 with inert fluid of density ρ_2. It will be assumed that the propellant reacts completely and comes to thermal equilibrium with the inert fluid in the combustion chamber.

The total mass flow which results from a constant volume flow V_0 may be expressed in terms of propellant density, the mass ratio K, and the density ratio β.

$$\dot{V}_0 = \frac{\dot{m}_1}{\rho_1} + \frac{\dot{m}_2}{\rho_2} = \dot{m}_1\left(\frac{1}{\rho_1} + \frac{K}{\rho_2}\right)$$

$$\dot{m}_1 = \frac{\dot{V}_0\rho_1}{1 + K/\beta}$$

$$\text{Total } \dot{m}_0 = \dot{m}_1 + \dot{m}_2 = \dot{m}_1(1 + K)$$

$$\dot{m}_0 = \dot{V}_0\rho_1\,\frac{(1 + K)}{(1 + K/\beta)} \qquad [1]$$

Note that K/β is the volume ratio; additive to propellant.

The static equilibrium temperature of the mixture of inert additive and active propellant is assumed given by the calorimetric relation

$$m_1c_1T_0 = (m_1c_1 + m_2c_2)T_m$$

or

$$T_m = T_0/(1 + K\alpha) \qquad [2]$$

The average specific heat $\bar{c}$ of the reacting mixture, assuming constant individual specific heats, can be expressed (from the definition) as

$$\bar{c} = \frac{c_1(1 + K\alpha)}{1 + K} \qquad [3]$$

If the mixture in the combustion chamber expands completely and adiabatically into a vacuum, the exhaust velocity u obtained will be proportional to the square root of the enthalpy change, or

$$u = \sqrt{2\bar{c}T_m} \qquad [4]$$

Influence of Variable Density on Thrust

We are now able to calculate the thrust as a function of mass ratio K and density ratio β. The thrust F_m of the fixed propellant flowing through an ideally expanded nozzle in vacuum will be, using Equations [1] through [4]

$$F_m = \dot{m}_0u = \dot{m}_0\sqrt{2\bar{c}T_m}$$

or

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[1] Staff Engineer. Member ARS.

[2] "Some Possibilities for Rocket Propellants," by A. S. Leonard, JOURNAL OF THE AMERICAN ROCKET SOCIETY, Dec. 1946, no. 68, p. 12.

$$F_m = \dot{V}_0\rho_1 \sqrt{2c_1T_0} \cdot \frac{\sqrt{1+K}}{1+K/\beta} \quad \ldots\ldots [5]$$

and

$$\frac{F_m}{F_1} = \frac{\sqrt{1+K}}{(1+K/\beta)} \quad \ldots\ldots [6]$$

where $F_1 = V_0\rho_1\sqrt{2c_1T_0}$, the thrust when pure propellant is used alone (i.e., $K = 0$, $K/\beta = 0$)

It is interesting to examine the behavior of Equation [6] as density ratio β and mass ratio K vary. We see that if

$$\beta \ll K, \text{ then } F_m < F_1$$

and if

$$\beta \gg K, \text{ then } F_m \to (\sqrt{1+K})F_1$$

Thus certain regimes of β and K must exist in which the *thrust* of a pure propellant can be improved upon by substitution of an inert but dense fluid for part of the propellant, maintaining total volume constant. The optimum value of K which results in maximum improvement of thrust (maximum F_m/F_1) is obtained by differentiating [6] with respect to K, giving

$$K = \beta - 2 \quad \ldots\ldots [7]$$

Although an optimum value of K (with β fixed) exists, there is no optimum value of β with K fixed. The best value for β is simply the largest one possible. In a practical application of course, β would be fixed by the available fluids. β would be the independent variable, and K a continuously variable dependent quantity.

The curves of Fig. 1 give the values of F_m/F_1 for K ranging from $^1/_4$ to 15, and for β ranging from $^1/_4$ to 12. From these it appears, for example, that thrust can be increased 51 per cent by filling the tank $^6/_7$ full by weight ($^3/_7$ by volume) with an inert fluid which is 8 times as dense as the original propellant.

The optimum volume ratio K/β corresponding to optimum mass ratio K is

$$\text{opt. vol. ratio} = \frac{K_{\text{opt.}}}{\beta} = \frac{1}{\beta}(\beta - 2) = 1 - \frac{2}{\beta} \quad \ldots\ldots [8]$$

As can be seen from either Equation [7] or Equation [8], the lowest useful additive density is about $\beta = 3$, for which the tank is $^1/_4$ full of additive. As β increases, the optimum volume ratio increases, so that at $\beta = 8$, for example, the tank is $^3/_7$ (43 per cent) full of inert fluid.

Influence of Variable Density on Missile Acceleration

The discussion up to this point has shown that *thrust* may be increased by inert additives, and since a constant dV_0/dt and tank volume have been assumed, this enhanced thrust will be exerted for the same duration as the original thrust. However, the criterion for vehicle performance is *acceleration*, since with fixed duration this determines terminal velocity. If we assume a vehicle the mass of which is nearly all propellant, we may readily calculate the initial acceleration (to which all subsequent values of acceleration will be linearly proportional) for say, vertical drag-free flight.

$$a_0 = \frac{F - gm_0}{m_0} = \frac{\dot{m}u}{m_0} - g \quad \ldots\ldots [9]$$

From [5]

$$a_0 = \frac{\dot{V}_0\rho_1\sqrt{2c_1T_0}}{m_0} \cdot \frac{\sqrt{1+K}}{1+K/\beta} - g$$

After integrating [1]

$$m_0 = V_0\rho_1 \cdot \frac{1+K}{1+K/\beta}$$

Thus

$$a_0 = \frac{\dot{V}_0}{V_0}\sqrt{\frac{2c_1T_0}{1+K}} - g \quad \ldots\ldots [10]$$

We see from [10], therefore, that any value of $K > 0$ will *decrease* the initial (and all later) values of acceleration, regardless of β, and hence that a dense inert additive is not helpful in the case of missile flight.

Influence of Variable Density on Assisted Take-Off of Aircraft

In the case of an assisted take-off unit applied to a large airplane, however, the mass of the propellant would be only a small part of the total mass, in which case the disadvantage of increased propellant density would not necessarily outweigh the benefit of increased

FIG. 1. EFFECT OF ADDING AN INERT FLUID TO THE PROPELLANT ON THE THRUST OF A LIQUID PROPELLANT ROCKET. F_m/F_1 = RATIO, THRUST WITH MIXED PROPELLANT TO THRUST WITH PURE PROPELLANT. $K = m_2/m_1$ = RATIO, MASS OF INERT ADDITIVE TO MASS OF PURE PROPELLANT. $\beta = \rho_2/\rho_1$ = RATIO, DENSITY OF INERT ADDITIVE TO DENSITY OF PURE PROPELLANT.

thrust. We may consider, for example, an aircraft of initial mass $NV_0\rho_1$; i.e., N times heavier than an assisted take-off rocket of volume V_0 filled with pure propellant. If, in Equation [9], one allows $g = 0$ (horizontal acceleration only) and adds to m_0 the mass $NV_0\rho_1$, one may by straightforward algebra arrive at an expression for a_m/a_1, the ratio of the initial acceleration a_m of the plane with a take-off rocket utilizing an inert admixture of ratio K, to the initial acceleration a_1 with a rocket of equal volume utilizing pure propellant. Assuming that a variety of dense additive fluids are available, and that optimum mass ratio $K = \beta - 2$ is used, whatever the value of β, the expression for a_m/a_1 may be constructed as shown in the following table:

	Pure Propellant (1)	Mixture (m)
Thrust (Eq. [5])	$\dot{V}_0\rho_1\sqrt{2c_1T_0}$	$\dot{V}_0\rho_1\sqrt{2c_1T_0}\cdot\dfrac{\beta}{2\sqrt{\beta-1}}$
Initial mass ATO unit (Eq. [1])	$V_0\rho_1$	$V_0\rho_1\cdot\dfrac{\beta}{2}$
Initial mass ATO unit plus aircraft	$V_0\rho_1(N+1)$	$V_0\rho_1\left(N+\dfrac{\beta}{2}\right)$
Initial acceleration	$\dfrac{\dot{V}_0}{V_0}\cdot\dfrac{\sqrt{2c_1T_0}}{N+1}$	$\dfrac{\dot{V}_0}{V_0}\cdot\sqrt{\dfrac{2c_1T_0}{\beta-1}}\cdot\dfrac{1}{1+2N/\beta}$

We see from this tabulation that the ratio of acceleration a_m with mixed propellant to acceleration a_1 with pure propellant is

$$\frac{a_m}{a_1} = \frac{N+1}{\sqrt{\beta-1}\left(1+\dfrac{2N}{\beta}\right)} \qquad [11$$

TABLE 1 TABLE OF ACCELERATION IMPROVEMENT a_m/a_1 OBTAINABLE IN ASSISTED TAKE-OFF UNITS

N \ β	3	5	8	17
5	0.98	1.07	1.19	1.42
10	1.02	1.15	1.32	1.67
20	1.04	1.19	1.40	1.86
50	1.05	1.23	1.47	2.01

We see from Table 1, for example, that if $\beta = 8$ ($K = 6$), which from Fig. 1 results in 51 per cent increase in thrust, the initial acceleration of a plane for which $N = 50$ can be increased 47 per cent over the acceleration when pure propellant is used.

Conclusions

In principle, the thrust of a constant-volume rocket may be increased by substituting for propellant appropriate amounts of a very dense inert fluid. This will not necessarily increase the acceleration of a vehicle driven by this rocket unless a sufficiently large fraction of the vehicle mass is fixed (not expendable).

It should be pointed out that a number of idealistic assumptions have been made, such as complete expansion into vacuum (Equation [4]), constant specific heats, and negligible structural mass (Equation [9]). The calculations may be considered therefore to give trends rather than engineering data.

Letters to the Editor*

Comment on "Heat Flow in Composite Slabs" by E. Mayer[1]

JACK LORELL[2]

Jet Propulsion Laboratory
California Institute of Technology, Pasadena, Calif.

In the course of his paper Mayer devotes some space to introducing the functions $\varphi_n(x)$, with

$$\varphi_n(x) = \begin{cases} \sqrt{\rho_1 c_1}\ f_{1n}(x) & -d_1 \leq x < 0 \\ \sqrt{\rho_2 c_2}\ f_{2n}(x) & 0 \leq x \leq d_2 \end{cases}$$

and proving the $\varphi_n(x)$ orthogonal. It may be of interest to point out that with some saving of effort, this orthogonality proof may be by-passed.

Essentially the difficulty arises because the dependent variable in Mayer's analysis is not well behaved at the interface, $x = 0$. If $h(x)$ represents either $f_1(x)$ or $f_2(x)$, according to whether x is less than or greater than zero, then dh/dx is not continuous at $x = 0$. As a consequence, the differential equation in question is not of Sturm-Liouville type, at least in regard to the type of solution desired. Mayer gets around this difficulty by deriving his result directly without recourse to Sturm-Liouville theory. However, the change of variable

$$y = \begin{cases} k_2 x \text{ for} & -d_1 \leq x \leq 0 \\ k_1 x \text{ for} & 0 \leq x \leq d_2 \end{cases}$$

makes it possible to use this theory.

The procedure would be to use the development as Mayer gives it up through his Equation [11″]. Then, noting that the h_n's are not orthogonal, the independent

(*Continued on page 222*)

Received April 30, 1952.

[1] JOURNAL OF THE AMERICAN ROCKET SOCIETY, vol. 22, May–June, 1952, pp. 150–158.

[2] Research Engineer, Theoretical Analysis Section.

* This section of the JOURNAL is open to letters not exceeding 600 words in length (or one and one-half columns) devoted to brief research reports or technical discussions of papers previously published. Such letters are published without editorial review, usually within two months of date of receipt. Style and manner of submission are the same as for regular contributions. (See inside back cover.)

Jet Propulsion News

C. F. WARNER, Purdue University, *Associate Editor*

with the assistance of W. G. BOHL

Rockets and Missiles

THE complete flight of a guided missile may now be followed through use of a "chain" radar system developed by the Army Signal Corps. This system also makes it possible to obtain accurate ballistic data along the missile's flight path.

The radar chain, located at the White Sands Proving Grounds, Las Cruces, New Mexico, consists of several stations spread along the range. Master stations are located at the major missile launching areas and tracking is initiated simultaneously with the launching of a missile. As the missile travels along its course, data from the master station are sent to the first relay station, automatically positioning the radar set at that station so that the set begins tracking the missile as soon as it comes into range. The process is repeated from station to station.

Data from all stations are automatically and instantaneously relayed back to the master station, enabling observers to watch flight characteristics of the missile. Range control officers maintain watch on the instruments to assure that the missile is following its flight path. Photographic records are made of the radar dials and oscilloscopes. At the terminal end of the missile's flight, radar automatically trains cameras with telescopic lenses on the missile as it dives into the target, providing a record of this vital phase of the missile trajectory.

* * *

JAMES H. WYLD, a pioneer in the development of regeneratively cooled rocket motors, presented his ideas of a "Model T space ship" at a recent meeting of the New York Section of the ARS. Before building a large space ship, Wyld suggested that a smaller Model T version should be constructed. The ship, designed to circulate in an orbit about 200 miles from the earth, would be a three-stage affair with the first two stages used to gain altitude and then jettisoned. The complete ship would have an over-all weight of 12.7 tons and a final weight of one-half ton after dropping tanks and motors. Wyld's ship would carry a pay load of only one-half ton including two lightweight men, their equipment, air supply for two days, automatic pilot equipment, and wing-equipped cabin. The skin of the space ship would have to be "of advanced design" to withstand the extreme temperatures encountered in descent and yet attain the necessary lightweight.

* * *

FIG. 1 shows the Oerlikon guided missile in flight. Unfortunately no performance data have been released to date. Some of the characteristics of this weapon were reported on p. 39 of the January–February issue of this JOURNAL.

* * *

THE new Lockheed F-94C, the North American F-86D, and the Republic F-84F are to be armed with increasing numbers of rockets rather than with additional conventional .50-cal machine guns. The armament will include the U. S. developed 2.75 in. rocket and the Swiss Oerlikon 8-cm rocket. As soon as possible the Oerlikon rocket will be obtained from the American Oerlikon Company which has broken ground for a new plant near Asheville, North Carolina.

* * *

ACCORDING to Assistant General Manager Leston P. Faneuf, of Bell Aircraft, Bell's Niagara Falls plant will be devoted almost entirely to the production of guided missiles by January 1953.

* * *

THE proposed U. S. military budget for the next fiscal year allocates more than twice the amount spent last year for numerous guided missile and rocket projects.

* * *

A 16 Kva water-cooled alternator for guided missile aircraft has been developed by Jack and Heintz, Inc. Because of the excessive heating problem encountered in the operation of this 400-cycle, 12,000-rpm unit, conventional cooling by air blast was not possible at

FIG. 1. OERLIKON ANTIAIRCRAFT GUIDED MISSILE IN POWERED FLIGHT

altitudes above 35,000 ft. The cooling problem was solved by injecting water through the alternator's hollow shaft and spraying it through four openings against the internal surfaces of the alternator. The water evaporates and cools these surfaces. The resulting steam is exhausted through a port. Three and one-half pounds of water are required per hour.

* * *

THE increased use of liquid rockets for missile and aircraft propulsion has created a demand for propellant servicing equipment. Republic Aviation Corporation has constructed a special truck designed to supply liquid oxygen and a mixture of water and alcohol to rocket aircraft. The $12^1/_2$-ton truck is equipped with a 900-gal, double-walled liquid oxygen tank and a 700-gal water-alcohol tank. Additional apparatus consists of fire extinguishing equipment, a compressor, pumps, and propellant lines. A two-man crew can operate the tanker.

Radio Transmission

LONG-distance communication may be revolutionized by the discovery of a new way of sending radio signals. If this method proves successful, it may be possible to apply it to the guidance and control of long range missiles and possibly satellite vehicles. The transmission method, developed by scientists from several universities and the National Bureau of Standards, in co-operation with the Collins Radio Company of Cedar Rapids, produces signals that apparently are not subject to fading and disappearance. Signals at a frequency of 49.8 megacycles have been transmitted from Cedar Rapids over a distance of 800 miles to Sterling, Virginia, irrespective of season, time of day, or atmospheric conditions.

A formal paper describing this discovery is scheduled to appear in the *Physical Review* published by the American Physical Society. Essentially the discovery is based upon a new concept of the nature of the ionosphere comprised of layers of ionized gases extending some 200 miles above the earth. Technically the layers are known as the E layer, the F_1, and F_2 layers.

A radio signal transmitted upward is presumed to hit one of these layers and may be absorbed, bent back, or just go into outer space. The frequency or length of the wave influences its ultimate destination. Long-distance transmission waves having frequencies between 5 and 20 megacycles are bent around the curvature of the earth by the ionosphere. Ionospheric disturbances may disrupt transmission at these frequencies for days.

Waves used for present-day television transmission are of higher frequency; therefore, they penetrate the ionosphere and are lost in outer space. It has been assumed that without the ionospheric reflection ultrahigh-frequency signals were limited to the optical lines of sight or the horizon. For many years it was known that amateur radio operators have on occasion maintained two-way communication over substantial distances, and television set owners have been surprised occasionally in getting an image from a station many miles away. These cases have been unpredictable and have been attributed to patches of relatively dense ionization of the lower atmosphere.

The scientists responsible for the new propagation method said that under the old theory it was assumed that most of incident intensity went through the ionosphere. Under the new theory so much energy is transmitted that enough of it returns to the ground to be received in strength at certain points. Using a diamond-shaped rhombic antenna, each leg of which was 500 ft long, the Collins Radio plant has been able to send a signal at a frequency of 49.8 megacycles with a power of 23 kilowatts for a distance of 1200 miles.

Foreign Developments

SOME measure of the progress achieved by Russia in the field of jet propulsion was revealed in a recent article by Jan C. Morton, published in *AVRO Canada News*. The first Russian jet plane, the MiG-7 (a MiG-3 air frame powered by a modified German Jumo 004 turbojet) flew in the spring of 1945. Data obtained from this experimental plane were used in the design of the MiG-9 twin-engine jet fighter. During 1947, 30 Rolls-Royce Derwent 5's and 25 Nene 1's were received from England. Under the direction of designer N. B. Chelong, the thrust of the Nene was increased from 4500-lb to 5500-lb by 1951. The new version of the Nene, the M-45, is currently being modified to produce between 6000- and 6500-lb static thrust.

Captured German axial flow units, the BMW-003A-2 and the Jumo 004B-4, were the basis for developments leading to the Russian M-003E and M-004H. The outputs of these engines were increased from 1760 lb- and 1980-lb thrust to 3750-lb and 4000-lb, respectively. Present production models known as M-012 and M-018 produce 6600-lb and 8000-lb static thrust.

The Russian designers have improved the performance of the German Walter HWK509C liquid fuel rocket which developed 4400-lb thrust in 1945 until in 1951 their version of this power plant developed 6600-lb thrust. This unit has undergone extensive flight tests in the Yak-21 experimental interceptor-fighter. The German BMW-718 (2500 lb thrust) liquid rocket assister has been improved and should compare favorably with the English Armstrong-Siddeley *Snarler*.

In addition to developing the well known MiG-15, the Russians have perfected the Yak-21, a version of the German Me-163, so that it can now attain a speed of over 900 mph. In 1951 the MiG-19 was introduced, a plane which closely resembles the German Ta-183. Several multijet-engined bombers are understood to be in production.

* * *

Reports have been received that the Russians are developing a giant long-range bomber similar in size

to our B-36. The bomber has very thin wings swept back at 35 degrees and a circular fuselage. Russian developed M-028 turboprop engines, developing 6900 hp and consisting of a 12-stage compressor and a 4-stage turbine, will power the plane. Auxiliary thrust is provided by Russian developed BMW-109-718 rocket engines having 6600-lb thrust. The bomber is reported to have a gross weight of 360,000 lb, a range of over 10,500 miles, and a speed of about 500 mph.

* * *

An announcement made by Prof. Kurt Tank, formerly of Focke-Wulf and now chief aeronautical consultant to General Peron, indicates that a four-jet delta-wing bomber is under construction in Argentina. It is assumed that this plane is a development of the Horten Ho XVIII B-2 since its designer, Dr. Reeniar Horten, was invited by the Air Ministry to complete his work in Argentina. The original design requested by the German Air Ministry was for a plane powered by 6 Jumo 004H turbojets, having a length of 62 ft, a span of 138 ft, with a speed of 560 mph and a service ceiling of 52,500 ft. Prof. Kurt Tank was recently reported to be traveling to Russia with his design for a multijet bomber.

* * *

The Leduc 021, fourth in a series of experimental ramjet powered aircraft, is nearing completion in France and is likely to fly this year. A Turbomeca Marbare turbojet mounted at each wing tip will make it possible for the 021 to take off unassisted from the ground. Performance figures indicate that with both turbojets and the ramjet power plants in operation the 021 should climb to an altitude of 32,800 ft in under two minutes and will have a duration of 15 minutes at 50,000 ft.

* * *

The French have equipped a DC-3 with an auxiliary Turbomeca turbojet and have recently completed a series of official tests under the direction of the French Air Ministry.

* * *

A delta wing fighter, the MD 550, is being designed by the Dassault firm of France for speeds up to Mach 1.5.

* * *

The Gemeaux IV, powered by a Turbomeca Aspin I ducted fan (440-lb thrust), has been flight tested in France. The light Gemeaux IV weighs 2640 lb with a useful load of 880 lb and has a span of 35.3 ft with an over-all length of 21.7 ft. It is expected that flight tests of the Gemeaux IV will demonstrate the successful application of jet propulsion to lightweight aircraft.

* * *

A. V. Roe Ltd. has a delta wing bomber under development. It is to be powered by four Bristol Olympus jet engines (approximately 13,000-lb static thrust each).

According to military experts, West Germany will soon re-enter the field of guided missiles. By the terms of the treaties signed on May 26 and 27, 1952, by the Foreign Ministers of West Germany, the NATO countries, and the United States, restrictions on German industry are to be largely eliminated, although special limitations on armament production are to remain in force. The manufacture of military aircraft is expressly forbidden. Jet aircraft for the 85,000-man German Air Force specified by the mutual defense treaty will probably be supplied by the United States. Published annexes of the treaty permit development and production of "short-range" guided missiles, but "long-range" missiles are forbidden. Drawing the line between the two categories is one of the problems to be resolved by NATO technical men. Ratification by the U. S. Senate and the other governments is expected soon.

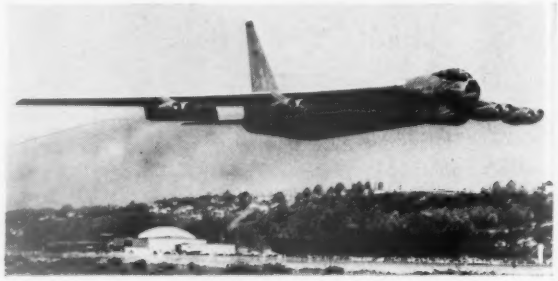


Fig. 2. Boeing YB-52 Stratofortress Takes Off on Maiden Flight, April 15, 1952

Aircraft

The U. S. Air Force has announced the general exterior details of its new long-range heavy jet bombers, the Boeing YB-52 and the Convair YB-60. The B-52 has a wing span of 185 ft, a length of 153 ft and a tail height of 58 ft. First flown on April 15, the B-52 has completed its first phase of testing and has been ordered into production. The larger YB-60 is 171 ft long and has a span of 206 ft. Both new bombers are powered by eight new Pratt and Whitney J-57 turbojets. (See Figs. 2, 3, and 4.)

* * *

On May 3 the British jet passenger liner Comet landed at Johannesburg after completing its flight from London. The British Overseas Airways Corporation said that the flying time was 17 hours and 16 minutes. The ship carried 36 passengers, 6 crewmen,

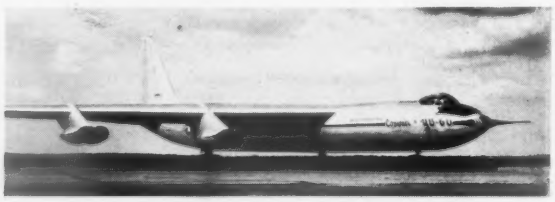


Fig. 3 Convair YB-60 Eight-Jet Bomber

and mail on this first scheduled jet air-transport civilian flight. The uniform passenger reaction was that they thoroughly enjoyed the fatigue-free flight because it was vibration-free. British officials said that they were at least four years ahead of the U. S. and other nations in the use of commercial jet aircraft. This summer thousands of Americans will have a chance to try out the new jet liner on the trans-atlantic run.

* * *

THE year 1952 promises to be a critical one for the U. S. air transport industry. Since the war, U. S. manufacturers have dominated the world civilian plane and engine field. This domination is now seriously threatened by the British. The production program for the Comet was recently doubled and orders have been placed by several European air lines. DeHavilland officials (makers of the Comet) predict that the sale of Comets and engines may amount to over "a thousand million pounds of export business over the next decade." As yet the U. S. does not have even the specifications for a turbojet airliner.

Turbojet Engines

THE Navy Bureau of Aeronautics has recently completed initial tests on a new powerful turbojet engine, an advanced version of the J-48 Turbo-Wasp built by Pratt and Whitney Aircraft. The new J-48 has passed its preflight tests and should be in production this fall.

* * *

PRATT and Whitney Aircraft is negotiating a license to permit the Ford Motor Company to build the axial flow J-57 Turbo-Wasp jet engine for military service.

* * *

THE Boeing lightweight gas turbine engine now is being manufactured on a production basis for the U. S. Navy.

* * *

A NEW slide rule with silhouettes and published information on the latest military aircraft has been devised by the General Electric Company Aircraft Gas Turbine Division. Information on air frames, operational data, and power plants of 42 aircraft are given.

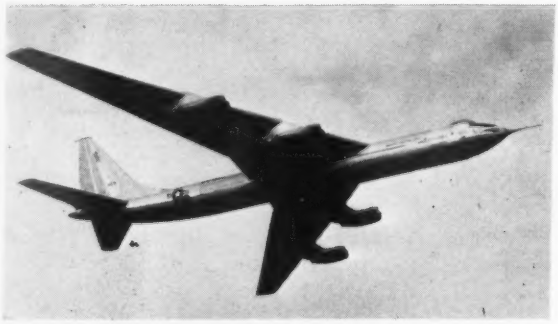


FIG. 4 FIRST FLIGHT OF CONVAIR YB-60, APRIL 18, 1952

FIG. 5 NORTHROP SCORPION F-89 ALL-WEATHER INTERCEPTOR

(Streaks of destructive fire lash out toward the target as a U. S. Air Force Northrop Scorpion F-89 all-weather interceptor turns loose the full power of six 20-mm cannon in this unique photograph, taken in the night sky over the Edwards Air Force Base firing range. Believed the first such night photograph ever taken, the picture shows the F-89 apparently suspended in the lacework of tracer fire from its own cannon. During the split-second exposure, the tracer streaks registered on the film before the F-89 was "frozen" by a powerful stroboscopic light flash, which lasted less than a $^{1}/_{5000}$ sec. America's most heavily armed fighters, the Scorpions are in service with the Air Defense Command, guarding U. S. borders against possible air attack. Advanced electronic equipment enables the F-89 to ferret out enemy airplanes as targets for its devastating cannon, even in bad weather or at night.)

New Materials

CLIMAXING two years of co-ordinated production and testing of ceramic-coated parts for high temperature applications, Ryan Aeronautical Company and California Metal Enameling Company have announced they have combined their facilities. The two firms disclosed that an improved ceramic, A-418-Ryanco-C has been developed as a result of laboratory and service tests. These tests show that some applications give better results with fairly thick coatings, while others are better when applied extremely thin. Not only do ceramic coatings extend the service life of jet engine alloy engine alloy parts but they also make it possible to use lower grade alloys. Many parts now made of Inconel may be fabricated from type 321 stainless protected with ceramic coatings. Their latest product is designed to perform under a continuous temperature of 1800 F.

* * *

A PROCESS for inseparably bonding aluminum to ferrous metals, making possible the production of units which combine the hardness and fatigue resistance of steel with aluminum's light weight and superior heat conductivity, has been perfected by Fairchild's Al-Fin Division. Although initially developed for heat transfer applications, this process is now used to produce bimetallic wings and fins for guided missiles.

AN "upsetting" machine made by the Utica Drop Forge and Tool Corporation is said to be able to produce gas turbine blades in one half the time formerly required. Since the costs of both the machine and its operation are low, the expense of producing turbojet blades may be reduced by over 50 per cent.

* * *

A QUARTZ paper developed by the Naval Research Laboratory can be manufactured cheaply and may be substituted for higher-cost asbestos insulating material. The new paper can withstand temperatures up to 3000 F making it much superior in this respect to asbestos. This paper may be used as an electric insulator.

Princeton Research Center Honors Late James V. Forrestal

The new James Forrestal Research Center of Princeton University, Princeton, N. J., was dedicated in colorful ceremonies on May 17, 1952. The principal speaker was Secretary of Defense Robert A. Lovett, and other addresses were given by Harold W. Dodds, president of Princeton University; Alfred E. Driscoll, governor of New Jersey; and Lewis L. Strauss, head of the Institute for Advanced Study at Princeton. Congratulatory telegrams were received from President Truman and former President Hoover.

President Truman's message said in part that "the United States Government will join with Princeton University in giving all possible assistance to the researchers at the Center who will be working to broaden our aeronautical knowledge. Their work will lead to valuable results by helping to strengthen United States defenses. It will also lead to developments which can benefit commercial aviation." President Dodds in his remarks noted that "although the Forrestal Research Center came into being at a time of national emergency and in some respects was the creature of that emergency, its long-range concern is the peacetime welfare of a peacetime people."

In a tour of the 850-acre establishment, 400 invited visitors inspected offices, classrooms, combustion laboratories, rocket motor test cells, wind tunnels, a helicopter laboratory, automatic computing equipment, chemistry laboratories, and other facilities. Graduate students and faculty members explained their research programs to the visitors. Research projects at the Center are directed by faculty members of the University, including: Prof. H. S. Taylor, chemistry; Prof. L. Lees, supersonics; Prof. L. Spitzer, physics; Prof. M. Summerfield, jet propulsion; Prof. A. A. Nikolsky, helicopters; Prof. R. N. Pease, chemical kinetics; Prof. L. Crocco, jet propulsion; Prof. R. H. Wilhelm, combustion; Prof. G. Akerlof, fuels chemistry; Prof. J. V. Charyk, rockets; and Prof. J. A. Wheeler, physics. Other members of the faculty are involved in other projects ranging from mathematics to biochemistry.

Daniel C. Sayre, director of the Center, announced the formation of a group of Corporation Associates, consisting of commercial organizations that have contributed financially to the endowment of the Center and are thus privileged to participate in the work of the Center. Among the Associates are: General Motors Corporation, The Firestone Tire and Rubber Company, International Business Machines Corporation, Union Oil Company, Consolidated Vultee Aircraft Corporation, and 15 other companies.

In addition to providing facilities for many research projects in jet propulsion and its related sciences, the the Forrestal laboratories also serve to house the post-graduate training program in jet propulsion sponsored by the Daniel and Florence Guggenheim Foundation and the adminstrative offices of Project SQUID, a Navy and Air Force jointly sponsored research program in combustion and jet propulsion.

New Facilities

GROUND has been broken for a new $6,000,000 steam- and gas-turbine development laboratory at the South Philadelphia works of the Westinghouse Electric Corporation. A staff of nearly 100 engineers will be employed. In co-operation with the U. S. Air Force, the laboratory will test a scale model of the giant air compressor which Westinghouse is building for a huge new wind tunnel being constructed for the Air Force's Arnold Engineering Development Center. This compressor will be the world's largest rotating machine and will supply air at supersonic speeds in the tunnel.

* * *

PHOENIX will become one of the West's major engine production centers when AiResearch Manufacturing Company of Arizona begins turning out gas-turbine power units. These units, used as turbojet starter and auxiliary aircraft power units, will be the major production item at the new $5,000,000 facility. Three gas-turbine test cells are among the laboratory facilities which will be utilized by AiResearch when production gets under way.

Personalities

W. W. BENDER has been appointed chief electronics engineer at Glenn L. Martin Company. In 1945 he was appointed technical director, pilotless aircraft section, and in 1948 was placed in charge of electromechanical design for missiles. His responsibilities were expanded in 1950 to embrace development of electromechanical systems for all Martin aircraft and missiles.

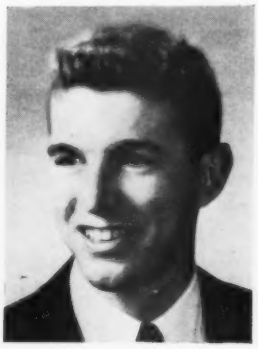
* * *

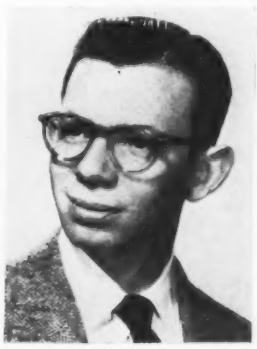
THE man who, while chief aerodynamicist at the Focke-Wulf aircraft factory in Germany, designed an airplane which many experts regard as the forerunner of the Russian MiG-15, is now heading a group of aeronautical engineers at the Glenn L. Martin Company. He is HANS MULTHOPP, who left Germany voluntarily in the spring of 1946 and went to England, where he joined the staff of the Royal Aircraft Es-

tablishment at Farnborough. He worked on experimental subsonic and supersonic projects there. Among his belongings brought over from Germany, Multhopp has a scale model of the jet fighter he designed for the Luftwaffe. It bears a striking resemblance to the MiG-15. Both Mr. and Mrs. Multhopp have filed first papers for American citizenship.

Daniel and Florence Guggenheim Jet Propulsion Fellowships Awarded to Sixteen Students

JOINT announcement has been made by Harry F. Guggenheim, president of the Daniel and Florence Guggenheim Foundation, Harold W. Dodds, president, Princeton University, and Lee A. DuBridge, president, California Institute of Technology, of the appointment of sixteen Daniel and Florence Guggenheim Jet Propulsion Fellows. The purpose of the Fellowships is to train young men for basic research and for leadership in the development of commercial and scientific applications of jet and rocket propulsion. The Fellows, selected for their technical ability, interest in rockets and jet propulsion, and demonstrated leadership qualities, will carry programs of advanced study and pioneering research at the Guggenheim Jet Propulsion Centers at Princeton University and California Institute of Technology.





ARS STUDENT MEMBERS WHO RECEIVED 1952 DANIEL AND FLORENCE GUGGENHEIM FELLOWSHIPS FOR ADVANCED STUDY IN JET AND ROCKET PROPULSION

(*Left to right:* Frederick Harshbarger, of North Liberty, Iowa, who will continue his studies at the Guggenheim Center, California Institute of Technology; George B. Milliard, of Kenmore, N. Y., and Walter R. Warren, Jr., of Long Island, N. Y., both chosen to study at the Guggenheim Center, Princeton; and Martin Goldsmith, of Pasadena, Calif., who will study at the Guggenheim Center, California Institute of Technology.)

Recipients of the grants for study at Princeton University are: Arnold Brooks, of Lynn, Mass., M.S. in aeronautical engineering, Columbia University, 1947; Ansel V. Gould, of Maplewood, N. J., B.S. in aeronautical engineering, Princeton, June 1952; Henry E. Hoercher, of Brooklyn, N. Y., B.S. in mechanical engineering, Polytechnic Institute of Brooklyn; Richard R. John, of New Rochelle, N. Y., M.S. in mechanical engineering, Princeton, June 1952; George B. Milliard, of Kenmore, N. Y., B.S. in mechanical engineering, Tufts College, June 1952, student member ARS; Edward T. Pitkin, of Putnam, Conn., B.S. in mechanical engineering, University of Connecticut School of Engineering; Walter R. Warren, Jr., of Rockaway Beach, N. Y., M.S. in aeronautical engineering, Princeton, June 1952, student member ARS.

The Fellowships for study at the California Institute of Technology are awarded to: Odus R. Burggraf, Jr., of Caledonia, Ohio, B.S. and M.S. in aeronautical engineering, Ohio State University, June 1952; Donald A. Dooley, of Long Beach, Calif., M.S. in aeronautics (jet propulsion option), Caltech, 1950; Martin Goldsmith, of Pasadena, Calif., M.S. in mechanical engineering, Caltech, June 1952, student member ARS; Denver C. Gore, Jr., of Portland, Ore., B.S. in aeronautical engineering, University of Washington; Frederick Harshbarger, of North Liberty, Iowa, majored in physics at Oberlin College, student member ARS; Charles A. Lindley, of Euclid, Ohio, B.S. and M.S. in aeronautical engineering, Ohio State University, 1949; Alphonse P. Peters, of Norwich, Conn., B.A. in chemistry, University of Connecticut, June 1952; Dean A. Rains, of Pasadena, Calif., M.S. in mechanical engineering, Caltech, 1951.

Renewals of Guggenheim Fellowships were granted to George E. Hlavka, of Racine, Wis., M.S. in mechanical engineering, Caltech, 1948, for further study at Caltech; and to Arnold Goldberg of Wahwah, N. J., and Marcel Vinokur, Long Island City, N. Y., for study at Princeton.

John Simon Guggenheim Memorial Foundation Fellowship Given for Rocket Navigation Study

PROF. Samuel Herrick, chairman of the Department of Astronomy, University of California, Los Angeles, Calif., has been awarded a John Simon Guggenheim Memorial Foundation Fellowship for his studies of extraterrestrial navigation: its present and prospective methods and instruments, and the effect of the introduction of rocket propulsion. The Fellowships are offered annually by the John Simon Guggenheim Memorial Foundation, 551 Fifth Avenue, New York,

N. Y., to further the development of scholars and artists by assisting them in research in any field of knowledge or in artistic creation in any of the fine arts.

Among Dr. Herrick's publications are the following articles: "Tables for Rocket and Comet Orbits," in *Astronomical Journal*, vol. 54, September 1949; "Space Rocket Trajectories," in *Journal of the British Interplanetary Society*, vol. 9, September 1950; "Rocket Navigation," in *Navigation*, vol. 2, December 1950; and "Interplanetary Navigation," idem., vol. 3, September-December 1951.

Comment on "Heat Flow in Composite Slabs"

(*Continued from page 215*)

variable, y, would be introduced, thus making the equations of Sturm-Liouville type[3] in h, and with weighting function $w(y)$ given by

$$w(y) = \begin{cases} \dfrac{\rho_1 c_1}{k_1 k_2^2} & -k_2 d_1 \leq y < 0 \\ \dfrac{\rho_2 c_2}{k_2 k_1^2} & 0 \leq y \leq k_1 d_2 \end{cases}$$

According to Sturm-Liouville theory, the h_n's are orthogonal with respect to the weighting function, i.e.,

$$\int_{-k_2 d_1}^{k_1 d_2} w(y)\, h_i(y)\, h_j(y)\, dy = 0 \qquad \text{for } i \neq j$$

Next, returning to the variable x, it is immediately inferred that Mayer's Equation [21] holds, i.e., that his functions φ_n are orthogonal.

It should be mentioned in regard to the use of the Sturm-Liouville theory here, that the boundary conditions of Mayer's problem do not imply Equation 8-28 on page 255 of Margenau and Murphy,[3] which is used to prove the Hermitian nature of the differential operator. However, this property does follow from the given boundary conditions, as can easily be checked.

Finally, it may be noted that some of the standard references, in dealing with Sturm-Liouville systems, consider only those for which the weighting function, $w(y)$, is continuous. At least as far as the orthogonality condition is concerned, this restriction is unnecessary.

[3] "The Mathematics of Physics and Chemistry," by H. Margenau and D. M. Murphy, D. Van Nostrand Co., New York, 1943, page 253.

OPPORTUNITY

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American Rocket Society News

H. K. WILGUS, *Associate Editor*

ARS Plans Active Program for Fall Meeting

On September 9–10, 1952, the American Rocket Society will hold its 1952 Fall Meeting at the Hotel Sheraton, Chicago, Ill., in conjunction with the Fall Meeting of The American Society of Mechanical Engineers, during the Centennial of Engineering.

The program will consist of three technical sessions at which papers will be presented on the latest developments in the jet propulsion and rocket technology fields, including the subject which has recently been receiving wide attention—interplanetary travel.

The program follows:

Session I

9:30 a. m., Tuesday, September 9:

Chairman: B. L. Dorman, Aerojet Engineering Corporation, Azusa, Calif.

Vice-Chairman: J. P. Layton, Forrestal Research Center, Princeton University, Princeton, N. J.

Upper Atmosphere Research, by Homer Newell, Naval Research Laboratory, Washington, D. C.

The High Altitude Sounding Rocket, by Milton Rosen and R. B. Snodgrass, both of the Naval Research Laboratory, Washington, D. C.

Exposure Hazards from Cosmic Radiation at Extreme Altitudes and in Free Space, by H. J. Schaeffer, U. S. Naval School of Aviation Medicine, U. S. Naval Air Station, Pensacola, Fla.

Session II

2:30 p.m., Tuesday, September 9:

Chairman: W. Proell, Standard Oil Company of Indiana, Whiting Research Laboratories, Whiting, Ind.

Vice-Chairman: N. J. Bowman, Standard Oil Company of Indiana, Whiting Research Laboratories, Whiting, Ind.

The Satellite Vehicle, by Wernher von Braun, Guided Missile Development Group, Redstone Arsenal, Huntsville, Ala.

Manned Flight at the Borders of Space, by Heinz Haber, Department of Engineering, University of California, Los Angeles, Calif.

Session III

9:30 a. m., Wednesday, September 10

Chairman: W. T. Olson, Lewis Flight Propulsion Laboratory, NACA, Cleveland, Ohio.

Vice-Chairman: John L. Sloop, Lewis Flight Propulsion Laboratory, NACA, Cleveland, Ohio.

Problems of Development of Ramjets for Supersonic Application, by J. O. Charshafian, Wright Aeronautical Division, Curtiss-Wright Corporation, Wood-Ridge, N. J.

The Pulsejet as a Present-Day Power Plant, by Brooks T. Morris, Marquardt Aircraft Company, Van Nuys, Calif.

On Stability and Control of Liquid Propellant Rocket Systems, by Y. C. Lee, Aerojet Engineering Corporation, Azusa, Calif.

Experimental Investigation of Performance of WFNA-JP-3 Rocket Motors at Different Combustion Pressures, by M. J. Zucrow and C. M. Beighley, both of the School of Mechanical Engineering, Purdue University, Lafayette, Ind.

Indiana Section Elects 1952–1953 Officers

On April 25, 1952, the members of the ARS Indiana Section held an election meeting in the Purdue Union Building, Purdue University, Lafayette, Ind.

The meeting was called to order by David McNay, president of the Section, and after a reading and approval of the minutes of the previous meeting, the following officers were elected for the year 1952–1953:

President, Delbert E. Robison; *Vice-President*, J. M. Botje; *Secretary*, Arthur B. Wiggins; *Treasurer*, Richard E. Watters; *Board of Directors*, Cecil F. Warner, W. L. Gilliland, Russell O. de Castongrene, David W. Schulz, Cecil L. Moore; *Ex officio member*, Dr. M. J. Zucrow.

Guided Missiles Subject of Gibson Lecture at Spring Banquet

The Indiana Section of the ARS and the Purdue University Chapter of the IAS met jointly for their spring banquet on May 22, 1952, in the West Faculty Lounge, Purdue Memorial Union Building, West Lafayette, Ind.

This dinner meeting highlighted the programs of the year for both organizations, and a large audience attended to hear Ralph E. Gibson, director, Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Md., speak on "The Philosophy of Guided Missiles."

Dr. Gibson came to Johns Hopkins in 1946 after having served as the first Director of Research at the Allegany Ballistics Laboratory, Cumberland, Md. The Applied Physics Laboratory at Johns Hopkins was instrumental in developing the proximity fuse, and since 1945 has been working on the Navy's Bumblebee Guided Missile Program.

Tschinkel Speaks on Liquid Propellants at Alabama Section Meeting

The Alabama Section of the ARS held its monthly meeting on April 22, 1952, in Huntsville, Ala., at which approximately 125 members and guests were present. Hans Hueter, vice-president, presided in the absence of the president, Martin B. Schilling.

The main business transacted was the appointment of committees to serve until the annual election in November. Committees were appointed as follows: *Membership:* Anthony Guzzo, chairman; Lawrence Nicastro, Karl Hager, Joseph Pelham, Hans Maus, and George Pedigo; *Program:* Joseph Wiggins, chairman; Eberhard Rees, Ludwig Roth, Lawrence Nicastro, Thomas Moore, Josef Michel, and George Henderson; *Publicity chairman:* Karl E. Katz; *Historian:* Miss Sarah Dishman.

Following the close of the business portion of the meeting, an illustrated lecture was given by J. G. Tschinkel, chief, Combustion and Fuels Section, Guided Missile Development Group, Ordnance Corps' Guided Missile Center, Redstone Arsenal, Huntsville, Ala.

Dr. Tschinkel discussed the properties which liquid propellants should have for optimum performance and the selection of oxidizers for application with liquid fuels. The talk was accompanied by slides showing comparative properties of suitable fuels and oxidizers. The speaker included in his talk various anecdotes of events which occurred during his service at the German rocket center at Peenemünde.

Concluding the lecture, Dr. Tschinkel showed several types of fuels for inspection by the group, and also performed several experiments which demonstrated the spontaneous reactions between certain fuels and oxidizers.

The next meeting of the Alabama Section has been scheduled for July 1, 1952, in Huntsville, Ala. The speaker for the evening will be W. H. Steurer, chief, Material Test Laboratory Section, Guided

Missile Development Group, Redstone Arsenal. Dr. Steurer will discuss metallurgy and alloys, especially those in high temperature ranges, as applicable to rockets and guided missiles.

Southern California Section Hears Talk on Ramjets

Another successful dinner meeting was held by the ARS Southern California Section on May 22, 1952, at the Pasadena Athletic Club, Pasadena, Calif.

Approximately 200 members and guests were present to hear Joseph Friedman, technical assistant to chief designer, Marquardt Aircraft Company, Van Nuys, Calif., deliver a talk on "Consideration of the Ramjet as an Aircraft Power Plant."

The subject aroused considerable interest, and, according to E. G. Crofut, member of the Section's Board of Directors, numerous requests have been received for further discussions in the air-breathing power plant field.

Exploration of Space Discussed by A. C. Clarke, BIS Chairman

The ARS New York Section, at its monthly meeting on May 22, 1952, was fortunate in having as its guest speaker, Arthur C. Clarke, chairman of the British Interplanetary Society, Fellow of the Royal Astronomical Society, and member of the Council, British Astronomical Association.

Mr. Clarke began his talk by recounting the early days of the BIS, which was founded in 1923. Despite much early ridicule, and later wartime suspension, the Society has grown to some 1800 members and enjoys widespread prestige as the largest organization in the English-speaking world dedicated primarily to the conquest of space.

The major part of Mr. Clarke's lecture was devoted to such topics as two- and three-stage rocket missiles, orbital satellite vehicles, "terrestrial," "lunar" (see Fig. 1), and "deep space" type rockets, lunar voyages, space suits, Moon and Mars bases, etc. His discussion was an illustrated résumé of the high points from his book, "The Exploration of Space," recently published by Harper & Bros. (see page 227 in this issue of the JOURNAL for a review of the book).

Both in his book and in his talk, Mr. Clarke emphasized the importance of satellite vehicles, particularly space stations. The assembly of such a station is seen in Fig. 1. One of the major uses of the orbital satellite station would center around its refueling capabilities. A rocket, taking off from the earth, would reach the station with its propellants all but expended. There, on the space station, it could refuel, or its crew could transfer to another type rocket especially designed for interorbital airless travel, and hence possessing no particular aerodynamic shape. In addition to its refueling capabilities, the satellite vehicle might well base radio and television stations, astronomical observatories, physics, chemistry, and biology laboratories, and meteorological observation stations.

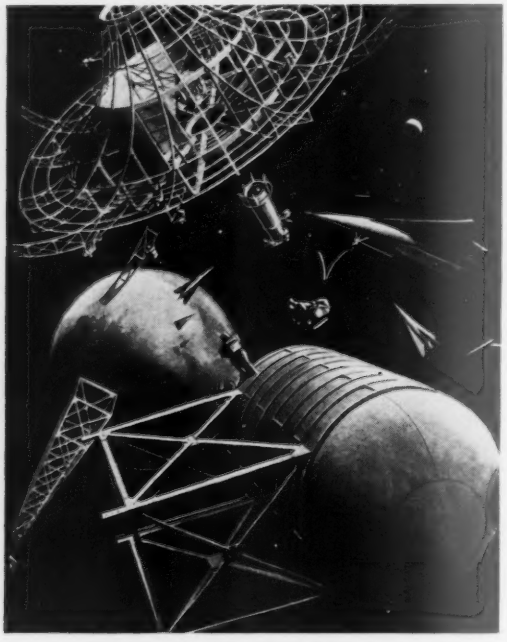

FIG. 1 ASSEMBLING THE SPACE STATION

(An illustration from "The Exploration of Space" by A. C. Clarke, chairman of the British Interplanetary Society. Mr. Clarke's speech before the ARS New York Section on May 22 was a résumé of the high points in his book.)

Washington-Baltimore Section Holds First Dinner Meeting

On May 27, 1952, the ARS Washington-Baltimore Section of the American Rocket Society held its first dinner meeting at the National Press Club in Washington, D. C. Approximately fifty members and guests were in attendance for the presentation of four short technical papers.

The meeting was opened by Harry J. Archer, president of the section. The four speakers were introduced by William L. Rogers of the Aerojet Engineering Corporation who was in charge of the arrangements for the meeting.

The first speaker was C. J. Libby of the Glenn L. Martin Company, who presented an interesting talk on the subject of "Zero Length Launchers for Guided Missiles." Mr. Libby's talk was accompanied by color movies showing tests of the Matador missile.

The second paper was presented by Mrs. I. D. Faro, who reported on work done by her and by F. K. Hill on "Hypersonic Wind Tunnel Research at the Applied Physics Laboratory of The Johns Hopkins University."

Complementing this paper and illustrating similarities and differences in facilities and techniques was a lecture by P. P. Wegener on "Hypersonic Wind Tunnel Research at the Naval Ordnance Laboratory at White Oak, Maryland."

The last paper on the program was in the nature of a challenge to the members of the rocket engineering profession. H. E. Newell, Jr., of the Naval Research Laboratory, presented a talk entitled "We Need Higher Altitude Rockets." Dr. Newell reviewed the development of rockets which had permitted the investigation of the upper atmosphere, and presented some of the data which had been obtained as a result of rocket firings. He showed that extension of our present knowledge concerning the upper atmosphere will be limited only by the heights to which rockets can be sent. Dr. Newell appealed to the rocket engineers to furnish the upper atmosphere scientists with the rockets needed for this task.

ARS Charter Presented to Northeastern New York Section

C. W. Chillson, President ARS, Guest Speaker

The ARS Northeastern New York Section held its dinner and installation meeting at the Hotel Van Curler, Schenec-

ARS CHAPTER PRESENTED AT NORTHEASTERN NEW YORK SECTION INSTALLATION MEETING MAY 13, 1952

F. M. Cooper (*left*), president, ARS Northeastern New York Section, receives the ARS Charter from Dr. R. W. Porter, member of the ARS National Board of Directors. C. W. Chillson (*center*), president ARS, was the guest speaker at the installation meeting.

tady, N. Y., on May 13, 1952. Over 80 members and guests attended to witness the presentation of the ARS charter and to hear the guest speaker, C. W. Chillson, president ARS, who spoke on "The Future and Potentialities of Rocket Propulsion."

In a ceremony preceding Mr. Chillson's address, Dr. R. W. Porter, member of the ARS National Board of Directors, presented the ARS Section Charter to F. M. Cooper, president of the Section.

In his talk, Mr. Chillson stated that the future and potentialities of rocket propulsion are not necessarily synonymous. Its potentialities are as limitless and boundless as space itself, he said, but its future is bounded by the use to which it is put. The present rocket field is a relatively small one with large responsibilities. The responsibility of members of the rocket profession is involved today almost exclusively with the military defense of democracies.

Eventually we may expect to witness the application of rocket propulsion as a means of space travel, Mr. Chillson remarked. This will probably require multistage units and will undoubtedly involve tremendous expenditures of time and money. Uncertainty in the performance of any of the vital functions, he pointed out, cannot be afforded.

Among the motivations that have been advanced as favoring the conquest of space, the speaker continued, are: (1) The expansion of inhabitable space; (2) prospecting and resource exploitation; (3) exploration and scientific curiosity; and (4) military exploitations.

Mr. Chillson warned the audience against becoming overoptimistic concerning the imminence of space travel. He pointed out the parallel between the promises of push-button warfare following World War II and those of interplanetary travel today. Many costly and ineffectual decisions were made in attempting to first implement the promises of push-button warfare and then to cover up the failure to meet the "too-much, too-soon" goals prophesied. Avoidance of the same mistakes, he said, is a matter of vital importance to the American Rocket Society.

Nominations Invited for ARS Awards

Members of the ARS in good standing are hereby invited to nominate one or more candidates for the awards listed below to be presented at the 7th Annual Convention of the American Rocket Society in New York, N. Y., November 30–December 5, 1952. Nominations must be received by the Secretary of the Society before August 15, 1952. Each nomination must be signed by the nominator and must contain the nominee's full name, his present title and position, and a brief account of the principal achievements on which the award is to be based.

The *Robert H. Goddard Memorial Lecture Award* shall be granted annually to the person or persons deemed by the Board of Directors to have made the greatest contributions during the year, or during a recent period including the current year, to the development of rocket propulsion in the liquid propellant field.

The *C. N. Hickman Award* shall be granted each year to the person who is deemed by the Board of Directors to have made the greatest contribution during the year, or during a recent period including the current year, to the development of rocket propulsion in the solid propellant field.

The *G. Edward Pendray Award* shall be granted each year to the person who is deemed by the Board of Directors to have made a major contribution to the technical literature on rocket propulsion or the more general field of jet propulsion. The Award may be granted either for a book or for an outstanding paper.

All nominations will be considered carefully by the Awards Committee, which is headed by Mr. Raymond Young, president, Reaction Motors, Inc. The selection of the award winners will rest entirely on the question of merit as judged by the Committee and will not be determined by the number of "votes" received. This procedure of soliciting nominations has been instituted in order not to overlook worthy candidates who may not otherwise come to the attention of the Committee.

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Third IAF Congress Meets in Stuttgart, Germany, Sept. 1–6

THE Third International Astronautical Federation Congress, to be held at Stuttgart, Germany, Sept. 1–6, 1952, will consist of a six-day program during which leading scientists from Great Britain, Austria, Germany, and the United States will present papers on astronautics, space travel, and related subjects.

The program from the Gesellschaft für Weltraumforschung, which is sponsoring the Congress, follows:

Sunday, August 31

6:00 p.m., Uhr, Schlossgarten Restaurant, Königstrasse 1, Am Hauptbahnhof, Nebenzimmer, Zwangloser Begrüssungsabend.

Monday, September 1

9–10:00 a.m., Eröffnung des Kongresses und der Ausstellung "Raketen und Raumfahrt," Presseempfang im Landesgewerbemuseum

10:30 a.m.–12:30 p.m., Erste Delegiertensitzung (geschlossene Veranstaltung, nach besonderer Tagesordnung).

2:00–6:00 p.m., Zweite Delegiertensitzung.

Park-Hotel, Villastrasse 21, Tagungssaal.

Tuesday, September 2

Park-Hotel, Villastrasse 21, Tagungssaal:

9:00 a.m.–12:30 p.m., Dritte Delegiertensitzung.

2:00–6:00 p.m., Vierte Delegiertensitzung.

Wednesday, September 3

8:00 a.m.–6:00 p.m., Schlossplatz (vordem Schloss). Tagesausflug. Abfahrt mit Autobussen in den Schwarzwald (Wildbad/Freudenstadt).

Thursday, September 4

Vorträge im Vortragssaal des Landesgewerbemuseums, Kienestrasse 18.

8:30–9:00 a.m., *Ing. J. Stemmer* (*SAA*): "Astronautische Grundlagenforschung auf internationaler Basis" (?)

September 4 (Cont.)

9:15–9:45 a.m., *Prof. Dr. F. Hecht* (*ÖGfW*): "Chemische Probleme des Weltraumflugs."

10:00–10:30 a.m., *G. V. E. Thompson, B.Sc., A.R.C.S.* (*BIS*): "Cost and Availability of High Energy Propellants."

11:00–11:30 a.m., *H. H. Kölle* (*GfW*): "Zur Bestimmung des optimalen Brennkammerdrucks bei R-Triebwerken."

11:45 a.m.–12:15 p.m., *Ing. G. E. Janzon* (*GfW*): "Umlaufende Flüssigkeitspumpe für Grossraketen."

2:00–2:30 p.m., *Ing. A. Hjertstrand* (*SIS*): "Grundsätzliches zur Frage des, unbemannten und bemannten Raumfahrzeugs."

2:45–3:15 p.m., Time reserved for AMERICAN ROCKET SOCIETY.

3:30–4:00 p.m., *Ing. H. Hoeppner* (*GfW*), *Dipl. Ing. H. Kühme*: "Die Weiterentwicklung der optimalen Lastrakete."

4:30–5:00 p.m., time reserved for AMERICAN ROCKET SOCIETY.

5:15–5:45 p.m., *Dr. H. Krause* (*GfW*): "Die Störungen d. Aussenstationsbah."

Friday, September 5

Vorträge im Vortragssaal des Landesgewerbemuseums, Kienestrasse 18:

8:30–9:00 a.m., *Dr. K. Wiese* (*Nwd. GfW*): "Das Problem der Strahlung im Weltraum."

9:15–9:45 a.m., *Dr. L. R. Shepherd* (*BIS*): "Possible Cosmic Ray Hazards."

September 5 (Cont.)

10:00–10:30 a.m., *Prof. Dr. J. Eugster* (*SAA*): "Die biologische Wirkung der kosmischen Strahlung, Methoden und neueste Ergebnisse."

11:00–11:30 a.m., *A. E. Slater, M.A., M.R.C.S., L.R.C.P.* (*BIS*): "Sensory Perceptions of the Weightless Condition."

11:45 a.m.–12.15 p.m., *H. K. Kaiser* (*Nwd. GfW*): "Physische Verhältnisse und Lebensbedingungen auf anderen Planeten."

2:00–2:30 p.m., *Dr. A. Meyer* (*GfW*): "Rechtliche Probleme des Weltraumflugs."

2:45–3:15 p.m., *Dr. R. Merten* (*GfW*): "Hochfrequenztechnische Probleme des Weltraumflugs."

3:30–4:00 p.m., *Dr. R. Horten* (*GfW*): "Zur Aerodynamik geflügelter Raketen" (?)

4:30–5:00 p.m., time reserved for AMERICAN ROCKET SOCIETY.

5:15–5:45 p.m., *Prof. H. Oberth* (*GfW*): "Gegenwärtige und zukünftige Probleme des Weltraumflugs."

8:00–9:00 p.m., *Prof. Dr. W. von Braun* (*GfW-ARS-BIS*): "Weltraumfahrt—eine internationale Gemeinschaftsaufgabe."

Saturday, September 6

9:00–12:30 p.m., Technische Werke, Lautenschlagerstrasse, Kinosaal Film-Matinée: "Raketen im Film."

7:00 p.m., Uhr Kursaal, Bad Cannstatt—Empfang der Kongressgäste durch die GfW und die Stadt Stuttgart.

NOTICE TO CONTRIBUTORS

Manuscripts submitted for publication *must* be double-spaced originals. There should be wide margins at both sides of the sheet and triple spacing around formulas to allow for the marking of directions for the printer.

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Only the simplest formulas should be typewritten, all others should be carefully written in pen and ink. The difference between capital and lower-case letters and Greek symbols should be clearly distinguished.

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Technical Literature Digest

H. S. SEIFERT, California Institute of Technology, *Associate Editor*

CONTRIBUTORS: D. Altman, D. I. Baker, R. B. Canright, F. C. Gunther, H. S. Seifert, F. H. Wright

Book Reviews

THE PLANETS, THEIR ORIGIN AND DEVELOPMENT, by Harold C. Urey, Yale Univ. Press, New Haven, 1952, xiii + 245 pp. $5.

Reviewed by CHARLES HEWITT DIX
California Institute of Technology

This remarkable study by Dr. Urey represents one of the most stimulating pieces of investigation that this reviewer has read for years. The author has waded into this problem and made what appears to be a real dent in it after a relatively short period of intensive study. The question of the origin of the solar system has challenged many of the best minds in the world; including Kant, Laplace, Poincaré, Jeans, Jeffreys, and others. Those who are convinced that scientific investigation should be applied only to problems of established solvability will do well to avoid this book! Others will find this book exciting reading, though never trivial or thin. Some background is necessary in the physical sciences, e.g., physics, chemistry, astronomy, and geology.

The first chapter, The Origin of the Solar System, reviews the current ideas of stellar evolution and the application of these ideas to the problem of the evolution of the solar system. The second chapter is a careful descriptive study of the features of the moon. The moon with no erosion agents presumably gives us good hints regarding collision craters for a great variety of objects, several of which appear to have been quite large. The moon can preserve very old evidence of impact craters. The earth is eroded.

The third chapter is a survey of evidence from the terrestrial planets with emphasis on the earth. Questions regarding the origin of the fluid core (known from seismology), the evolution of the atmospheres, and the main features of the bodies are discussed. The fourth chapter refers to chemical processes. Since chemical forces play a major part in the aggregation processes, questions of chemical thermodynamics play an important role. Some of the hypotheses of the past are shown to be highly improbable. The notion of a very hot origin is discarded for what appear to be good reasons. The fifth chapter refers to the very difficult problem of the earth's heat balance. A convective earth of high viscosity is tentatively accepted. The sixth chapter is on the abundance of the elements in the universe and the chemical composition of the solar system. The advances made in recent years in this part of geochemistry and astrophysics are ably reviewed, and their bearing on the general problem clearly discussed.

A summary chapter, appendices, and a good index are included. A very fine large photo of the moon is included as a folded plate.

The book represents a truly remarkable collection of a wide variety of kinds of data, of kinds of investigations, and of hypotheses. The greatest effort is made to push to conclusions along many different deductive lines and confront conclusions with observations and with other conclusions. As the author states, new facts and further work will alter many of the results of his work. No one can doubt that the author has made an important and, for future and present workers in the field, a highly stimulating contribution.

THE EXPLORATION OF SPACE, by Arthur C. Clarke, Harper & Bros., New York, 1952, xiii + 199 pp. $3.50.

Reviewed by F. C. DURANT, III,
Lt. Cmdr., U. S. Navy

Close upon the heels of "Interplanetary Flight," the author has published this new work which, although covering in more detail much the same ground, is destined, however, for a wider audience. The presentation is nontechnical but scientifically accurate within generally accepted limits. All known problems which will be encountered in space flight are treated with lucidity and freshness of approach. One feels that some subjects are treated with brevity because of press of space rather than competency of the author, an experienced young physicist turned writer.

The book contains 18 chapters. Commencing with a review of history of interest in space flight, both ancient and modern concepts are briefly noted. In successive chapters the principles of rocket propulsion and dynamics of "escape" and space flight follow. An introduction to astronomy is introduced to establish relative distances and to postulate what may be found at the end of a journey to the moon and planets. Next the artificial earth satellite (space station) as a key to interplanetary flight is developed. Finally, the author soars on to the possibility and ramifications of interstellar flight. Of particular interest is his treatment of the relativity aspects of flight approaching the speed of light and the resultant "stretching of time." To those unfamiliar with Einstein's theory of special relativity, this will be a new and intriguing phase of the subject. The book closes on a philosophical examination of the impact of the realization of space flight upon man and his culture.

The reviewer enjoyed "The Exploration of Space" immensely. Worthy of special mention are the fourteen well-chosen illustrations. Many are new, particularly the excellent drawings by R. A. Smith and the four four-color reproductions of paintings by Leslie Carr. This work is recommended for all libraries, personal and public, desiring basic works on space flight.

THE HANDBOOK OF MEASUREMENT AND CONTROL, edited by M. F. Behar, Instruments Publishing Co., Inc., Pittsburgh, Pa., 291 pp. (of which approximately 40% is advertising related to the text. Approximately 60% of the text was written by M. F. Behar, the remainder being contributed by twelve other authors).

Reviewed by R. STOTT,
Jet Propulsion Laboratory
California Institute of Technology

This handbook, as noted in the preface, is intended to bring information to the "instrument" man (a title rather indiscriminately applied to scientist, engineer, or application or maintenance man) concerning classification, operation, and applications of instruments. Instruments are defined as all indicating, recording, integrating, controlling, and computing devices used in measurement, inspection, testing, and control applications.

The contents consist of 16 chapters describing methods for the measurement or control of temperature, flow rate, pressure, speed, electrical quantities, physical properties and dimensions, chemical processes, and nuclear reactions, with special emphasis on the classification of systems and instruments. Because of the complexity, diversity, and rapid advancement of the field, it is a difficult, if not impossible, task to produce a handbook adequately encompassing the subject described; however, the contents should be of special interest to industrial instrument application or maintenance personnel, and many of the

systems and techniques described are of value in connection with instrument development problems.

PALOMAR, by Helen Wright, The Macmillan Co., New York, 1952, xi + 188 pps. $3.75.

Reviewed by HOWARD S. SEIFERT
Jet Propulsion Laboratory
California Institute of Technology

This well-written and elegantly illustrated little book is an anecdotal history of one of the most glamorous scientific adventures of an era distinguished for many technical achievements—the building of the 200-in. reflecting telescope at Mt. Palomar. It traces the progress of the telescope from the time it was an inspiration in the mind of George Ellery Hale, its noted sponsor, until it was dedicated in 1948. Much light is shed on the diplomatic maneuverings necessary to get the money for so large a project.

Miss Wright has been trained as an astronomer and has already achieved distinction as the author of "A Treasury of Science." She includes clear qualitative descriptions of the physical processes of the telescope and its uses, which are understandable to a general audience. Her material is authentically documented, being gathered in the course of the preparation of an official biography of George Ellery Hale.

It is interesting to speculate whether a similar history will some day be written of another stirring project—the space rocket —which today stands about where the 200-in. telescope did in 1920.

HIGH-SPEED PHOTOGRAPHY, vol. 3, published by the *Journal of the Society of Motion Picture and Television Engineers*, 1951, 159 pp., illus.

Reviewed by R. W. BOWERSOX,
Jet Propulsion Laboratory
California Institute of Technology

Volume 3 of this series on high-speed photography, like its two predecessors, is comprised of a number of short papers which have been presented at the conventions of the Society of Motion Picture and Television Engineers during the past two years. Since the first two volumes have discussed in some detail the various commercially available high-speed cameras as well as special cameras constructed for research in this country and abroad, this volume merely continues with the discussion of further camera and light-source development. The material is mainly descriptive in nature although it contains details of a number of the circuits used in synchronizing the cameras with the action to be photographed.

One of the three new cameras described in this volume is that developed by M. Sultanoff at the Aberdeen Proving Ground, Md., for the photography of self-luminous detonation and shock waves. Since these transients travel at a speed of about 25,000 ft/sec, the photographic exposures should not exceed 1/10 microsec. Mr. Sultanoff has constructed a camera capable of taking up to 300 frames at a rate of 100,000,000 pictures per sec. The picture is photographed through a moving grid composed of a large number of narrow parallel slits and must be viewed through the same set of slits. The final effect is similar to that used in certain animated greeting cards.

Perhaps of most interest to the readers of the ARS JOURNAL is the paper on photography in the rocket test program at Inyokern, Calif., which describes the Askania, Bowen, Mitchell, and Eastman high-speed cameras used in the photography of rocket trajectories. The Askania and Mitchell cameras track the missile throughout its range and give accurate data on position, roll, pitch, etc. The Bowen camera is nontracking and is therefore used on short trajectories. Interesting photographs are given of the various types of cameras in use in the field as well as sample results obtained from each camera.

Other papers of particular interest are one which describes the pressured ballistic range at the Naval Ordnance Laboratory and one on photography of underwater explosions at depths down to two miles. At the end of this volume is a complete bibliography on high-speed photography which will prove to be very useful to all workers in this field.

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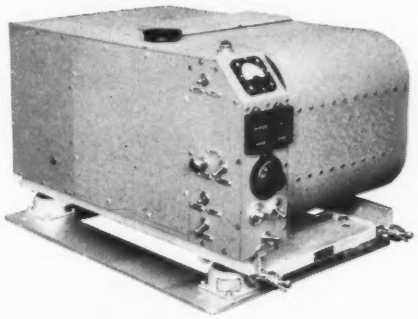

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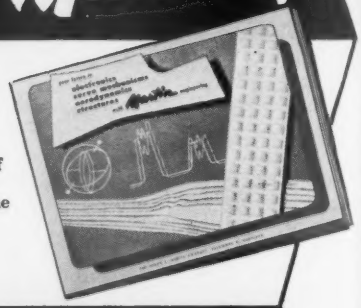

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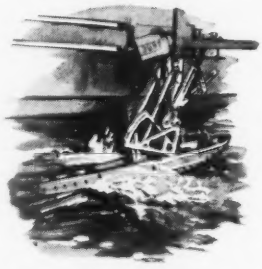

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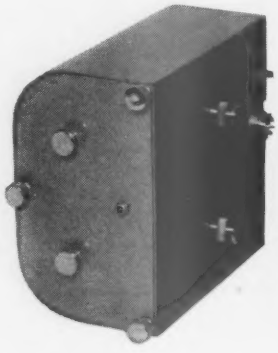

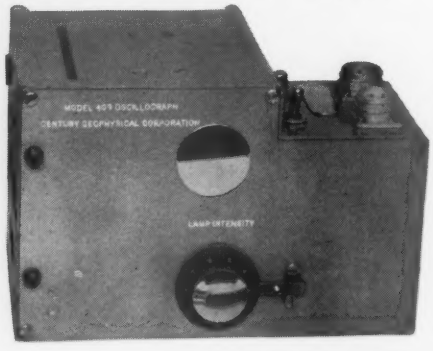

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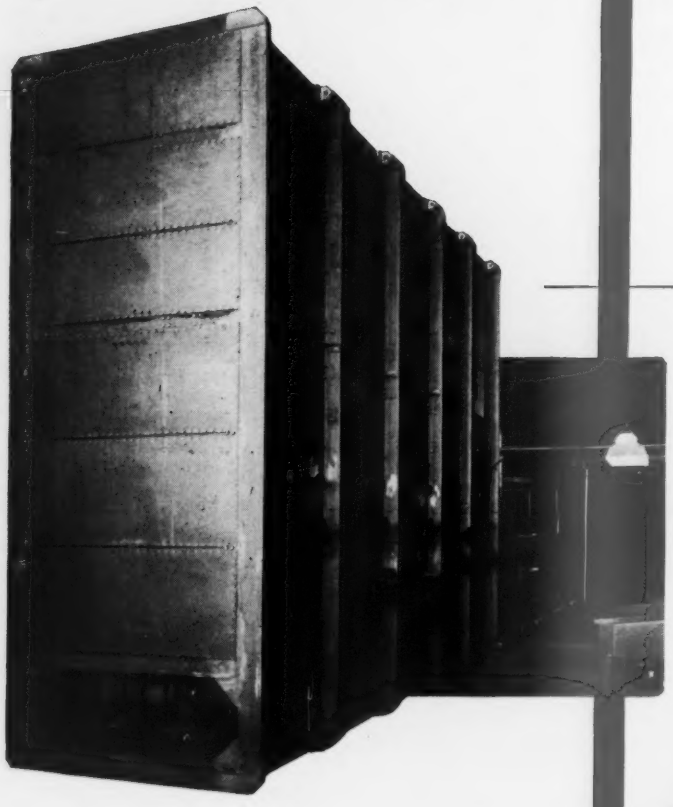

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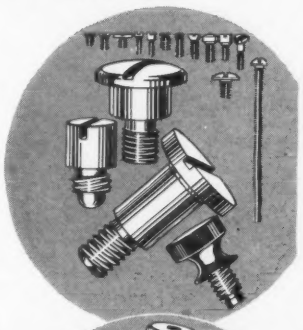

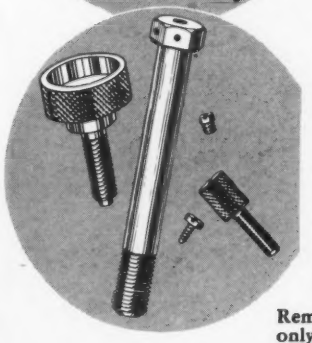

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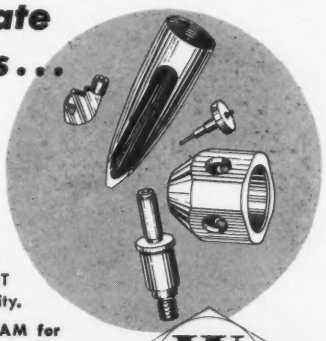

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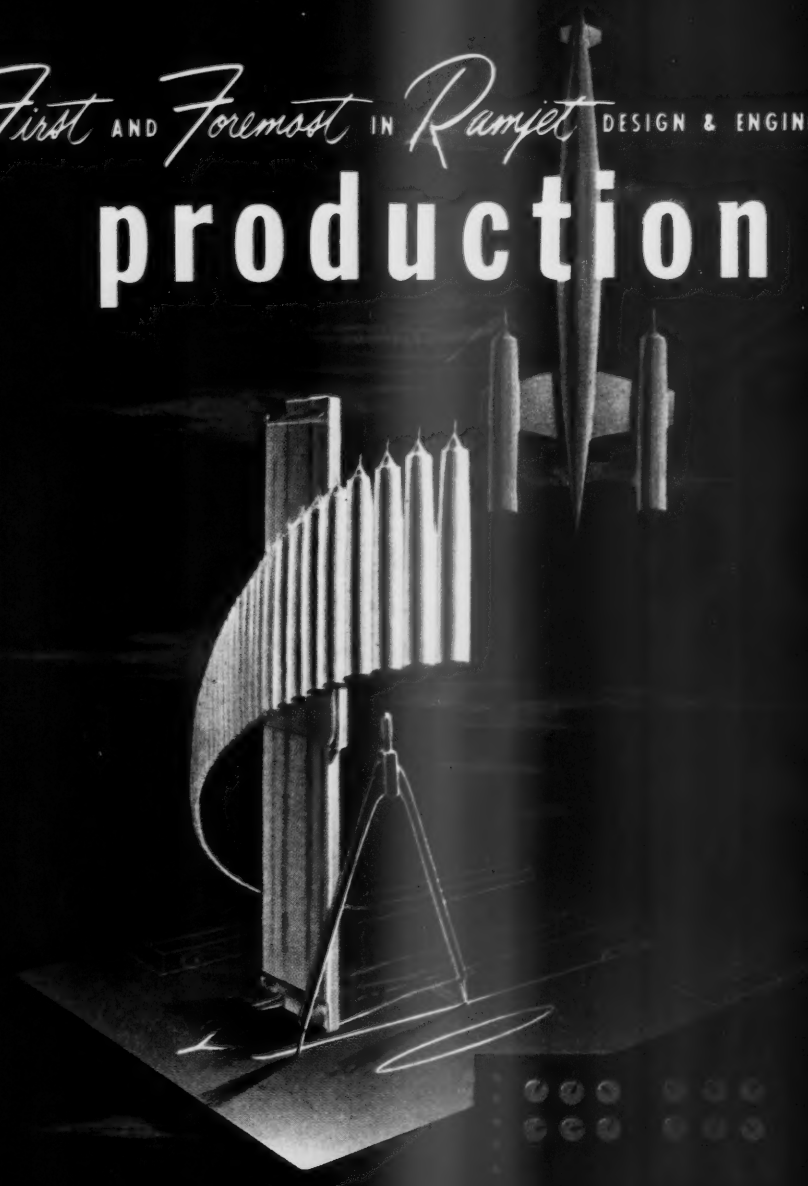

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